

Treatment of Measurement Error in Assessing Parameters of Multi-attribute Utility Functions¹

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Abstract

Most available methods for estimating parameters of multi-attribute utility models overlook presence of response error in the assessment phase. This paper is motivated by the fact that response error is present in all utility assessment procedures. Consequently, a suitable estimation method in this context should incorporate this fact, which is an intrinsic part of any assessment procedure. We propose a least-squares estimation method for estimating the relative weights of multi-attributes additive utility function. The method, which is based on decomposition assessment procedure, can accommodate presence of random error as well as systematic bias in assessment. Simulation studies demonstrate that the proposed estimation procedure will lead to correct estimation of parameters in a variety of situations. The proposed procedure for incorporating response error is quite general and can be used as a framework for estimating parameters of other multi-attribute utility functions.

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1. Introduction

Despite evidences against descriptive validity of expected utility (Kahneman and Tversky, 1979, Luce, 1988), multi-attribute utility theory (Keeny and Raiffa 1976; Von Winterfeldt and Edwards, 1986; Keeny, 1992), remains the most powerful prescriptive method for analyzing decision problems involving multiple objectives, particularly for analysis of preferences under certainty (Dyer and Sarin, 1979; Krantz, Luce, Suppes, and Teversky 1971; Miamoto, 1992).

If $x = (x_1, x_2, \dots, x_n)$ is a vector of attributes which describes a possible outcome, the utility function $U(x)$ describes a decision maker's preference ranking of all possible outcomes. What characterizes multi-attribute utility function is the attempt to specify the form of $U(x)$ in such a way as to elucidate the trade-offs among attributes. This is accomplished by adding to the basic set of axioms of von Neumann and Morgenstern (1947), some further assumptions about the decision maker's preference structure. One such assumption is that of mutually utility independence for a discussion of which we refer to (Keeny and Raiffa 1976). Under this assumption $U(x)$ may be written as

$$1 + KU(x) = \prod_{j=1}^n [1 + KW_j u_j(x_j)] \quad (1)$$

Where $0 \leq u_j(x_j) \leq 1$, K and W satisfy the constraint

$$1 + K = \prod_{j=1}^n [1 + KW_j]. \quad (2)$$

Furthermore, $K > -1$ and $1 > W_j > 0$.

When $K \rightarrow 0$, it can be shown that $U(x)$ in (1) tends to

$$U(x) = \sum_{j=1}^n W_j u_j(x_j). \quad (3)$$

Equation (3) is labelled as the linear additive function and corresponds to a particular case of utility independence called value independence (Keeny, 1974). The functions $u_j(x_j)$ are called univalued utility functions and W_j represent the relative weights reflecting contribution of each utility function to the overall utility function $U(x)$ and they shall satisfy the constraint

$$\sum_{j=1}^n W_j = 1. \quad (4)$$

Equation (3) is the most popular model used in practice and is commonly utilized in development of decision support systems (DSS). In this study we focus our attention on application of this additive function as presented in (3).

The assessment of the weights and individual utility functions can be either based on holistic or decomposition procedures. Both holistic and decomposition assessment methods for preference

assessment contain measurement error (Fischhoff, Slovic and Lichtenstein 1980). However, in most multi-attribute decision methods, presence of measurement errors in assessment procedure has been ignored. Laskey and Fischer (1987) contemplate this deterministic view and treatment of preference assessment to the fact that in Keeny and Raiffa (1976) expected utility theory, preferences are assumed to be deterministic and known with certainty. That is, the expected utility theory lacks an error theory. To overcome this problem, in practice, a variety of methods for checking consistency in preference assessment have been developed (e.g Keeny and Raiffa (1976) and Farquhar, 1984).

Etezadi-Amoli and Ciampi (1983), motivated by the work of Barron and Person (1979), introduced a general approach for modeling measurement error, and provided the SEE (Simultaneous Estimation of Everything) procedure for estimating parameter of Keeny and Raiffa (1976) multi-attribute utility theory (MAUT) models. The SEE procedure is developed to estimate parameters of Keeny and Raiffa linear or multiplicative models based on holistic assessment method. Laskey and Fischer (1987) explored the nature and extent of response error based on direct assessment. However, a mathematically rigorous but operationally simple and practical estimation method for parameter of MAUT based on decomposition assessment procedure has not been developed yet.

In the following, we first provide a measurement model for assessment of weights in linear MAUT function. Then, in section three, we introduce an estimation procedure for assessment of relative weights of the linear MAUT function. The estimation procedure introduced here is based on the least-square principles and assumes decomposition assessment procedure. In section four, a confidence interval for overall utility of multi-attribute outcomes is developed which is based on the assumption of known univalued utility functions. To evaluate performance of the proposed estimation method, in section five, we report results of some simulation studies. Finally, in section six, a discussion of the proposed method and its implication is provided.

2. A measurement model for the assessed weights

Using the classical measurement model (Lord and Novick 1974), taking a measurement i on a person results in a numerical value that we denote by y_i . This value (observed score) depends on a particular measurement situation, and is only one of the many possible values that may be obtained. That is, the measurement y_i is conceived as a particular realization of a random variable Y_i defined on the set of all possible values y_i that might be observed. The true score τ_i of a person on a particular measurement i is fixed and defined as the expected value of the observed score. That is,

$$\tau_i \equiv \mathcal{E}Y_i \quad (5)$$

where Y_i is the random variable taking values y_i . Furthermore, we define the error of measurement as the amount by which any observed value deviates from its corresponding true value.

$$E_i = Y_i - \tau_i \quad (6)$$

The random variable Y_i is assumed to be independent of one another. Consequently, E_i are independent of each other and $\mathcal{E}E_i = 0$. Thus, the classical measurement model for a particular measurement i may be written as:

$$Y_i = \tau_i + E_i \quad (7)$$

To develop a measurement model for assessed values of weights W_j , because of the constrain (4), the

weights will not be independent of one another. Consequently, if we assess each weight separately and model the measurement error based on equation (6), due to the above dependency, the error of measurement from such a process cannot be assumed independent.

To overcome this problem, instead of assessing weight of each attribute independently, we propose assessing the relative weight of each attribute with respect to the weight of one selected attribute, for example, W_n . We recognize that all assessments of relative weights (ratios), similar to any other measurements, are always subject to a response error. That is, the assessor cannot provide the true value of the relative weights $\gamma_j = W_j/W_n$.

Since these relative weights are always positive, we may represent them as an exponential function. Consequently the observed relative weights r_j (measured values), may be modeled as:

$$\ln(r_j) = \ln(\gamma_j) + f_j; j = 1, \dots, n-1, \quad (8)$$

where $r_j = w_j/w_n$ are the assessed (observed) ratios and $f_j, j = 1, \dots, n-1$, represent measurement errors and assumed to be normally distributed with equal variance. That is, $f_j \sim N(0, \sigma_f^2)$ for all j .

Equation (8) resembles the general measurement model presented in (7). However, to estimate the relative weights for an individual user, we require an estimate of the variance of errors which may be obtained by repeated measures of $r_j = w_j/w_n$.

To obtain repeated measures, we propose to measure first the relative weights of each attribute with respect to the weight of the most important attribute using a variation of the swing method. Then we will repeat the assessment procedure by measuring the relative weights or importance of each attribute with respect to the weight of the least important attribute.

The advantage of using the swing method is that it can provide directly measures of ratios. It is important to note that in using the swing method, one should not assess the relative weights without due consideration of the range of attributes involved in the model. (see e.g. Von Winterfeldt, Edwards, 1986).

Suppose the first set of assessments provides the relative weight of the attributes with respect to the most important attribute and suppose the most important attribute for the decision maker is x_n .

Let:

$$\ln(\gamma_j) = \ln(W_j/W_n) = h_j; j = 1, \dots, n-1. \quad (9)$$

Then we may rewrite (8) as:

$$\ln(r_j) = h_j + f_j; j = 1, \dots, n-1. \quad (10)$$

From (9) we obtain,

$$W_j = W_n \exp(h_j); j = 1, \dots, n-1. \quad (11)$$

From (4) and (11) we obtain,

$$W_1 + \dots + W_{n-1} + W_n = W_n (\exp(h_1) + \dots + \exp(h_{n-1}) + 1) = 1, \quad (12)$$

thus,

$$W_n = 1/(\exp(h_1) + \dots + \exp(h_{n-1}) + 1). \quad (13)$$

Suppose the least important attribute among those under consideration is the attribute x_1 . Suppose further that we obtain a second set of measurement of the relative weights for all attributes with respect to x_1 . Similarly to (8), we can write:

$$\ln(r'_j) = \ln(\gamma'_j) + f'_j ; j = 2, \dots, n. \quad (14)$$

where: $r'_j = w_j/w_1$ is the assessed relative importance of attribute x_j to x_1 , and $\gamma'_j = W_j/W_1$ the corresponding true relative importance.

To simplify the estimation procedure, we assume that dispersion of measurement errors for the two sets of assessments of relative weights are the same. That is, $f'_j \sim N(0, \sigma_f^2)$ for all j .

By dividing both the numerator and the denominator of γ'_j by W_n we obtain:

$$\gamma'_j = (W_j/W_n) / (W_1/W_n) = \gamma_j/\gamma_1. \quad (15)$$

or

$$\ln(\gamma'_j) = \ln(\gamma_j) - \ln(\gamma_1). \quad (16)$$

Substituting (16) into equation (14) yields

$$\ln(r'_j) = h_j - h_1 + f'_j ; j = 2, \dots, n. \quad (17)$$

3. Estimation procedure

In the followings a least-squares procedure for estimation of parameters of two different models will be presented.

3.1 Least squares procedure without intercept

The two sets of assessments obtained from the above procedures, i.e. (8) and (17), may be considered together in the following multiple regression model.

$$y = Dh + e. \quad (18)$$

Where y is a $2(n-1)$ column vector containing log of the assessed values of the two ratios r_j and r'_j ,

$$y' = [\ln(r_1), \ln(r_2), \dots, \ln(r_{n-1}), \ln(r'_2), \ln(r'_3), \dots, \ln(r'_n)].$$

D is a $2(n-1) \times (n-1)$ model matrix with constants equal to 0, 1 or -1 as demonstrated below. Note that the first $n-1$ rows of this matrix correspond to (8) and the remaining $n-1$ rows correspond to (17).

$$D = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 & 0 \\ 0 & 1 & 0 & \dots & 0 & 0 \\ \cdot & \cdot & \cdot & \dots & \cdot & \cdot \\ 0 & 0 & 0 & \dots & 0 & 1 \\ -1 & 1 & 0 & \dots & 0 & 0 \\ -1 & 0 & 1 & \dots & 0 & 0 \\ \cdot & \cdot & \cdot & \dots & \cdot & \cdot \\ -1 & 0 & 0 & \dots & 0 & 1 \\ -1 & 0 & 0 & \dots & 0 & 0 \end{bmatrix}$$

The $n-1$ dimensional column vector h contains parameters of the model, which are logarithm of the relative weights or ratios.

$$h' = [h_1, h_2, \dots, h_{n-1}],$$

and e is the random vector of error terms corresponding to the values of f_j and d_j .

$$e' = [f_1, f_2, \dots, f_{n-1}, f_2', \dots, f_n'].$$

With the above arrangement, one may simply use the ordinary least squares procedure to estimate parameters of the model.

3.2 Generalization

We can generalize this regression model by adding a constant term (intercept), to each of the two measurement models presented in (8) and (17).

$$\ln(r_j) = c1 + h_j + f_j; j = 1, \dots, n-1, \quad (19)$$

$$\ln(r'_j) = c2 + h_j - h_1 + f'_j; j = 2, \dots, n. \quad (20)$$

By adding these constant terms $c1$ and $c2$ to the above equations we will be able to incorporate bias in the assessment process. That is, if an assessor, in her/his first or second sets of assessments systematically over or under estimates the relative weights of attributes under consideration, the model would be able to accommodate such biases in assessments and correct them accordingly. These constant terms can be easily incorporated into the regression model (18) by adding two indicator (dummy) variables to matrix D ; where the first indicator represents the $n-1$ assessments corresponding to the first set of assessments and the second one to the remaining $n-1$ assessments.

Note that by adding these constant terms in the model we will loose 2 degrees of freedom. Thus, in

practice, to implement these corrections, the decision problem should involve at least 4 attributes. When the problem involves 4 attributes, we will have six measures and 5 parameters to estimate, 3 corresponding to estimation of h_j , and 2 parameters for the constant terms. Thus, we will have one degree of freedom left for the residuals. In special cases, when the bias in assessment of relative weights for the two procedures can be assumed to be equal, we will simply add a column of 1 to matrix D to account for this bias and gain an extra degree of freedom for the residuals.

4. Confidence interval for multi-attribute outcomes

Since $W_j = W_n \gamma_j$, from Eq. (3) the overall utility of a multi-attribute outcome x may be written as

$$U(x) = \sum_{j=1}^n W_j u_j(x_j) = W_n \left[\sum_{j=1}^{n-1} \gamma_j u_j(x_j) + u_n(x_n) \right] \quad (21)$$

Similarly we may write

$$1 - U(x) = \sum_{j=1}^n W_j - \sum_{j=1}^n W_j u_j(x_j) = W_n \left\{ \sum_{j=1}^{n-1} \gamma_j [1 - u_j(x_j)] + 1 - u_n(x_n) \right\} \quad (22)$$

Consequently, from (21) and (22) we have

$$\frac{\hat{U}(x)}{1 - \hat{U}(x)} = \frac{\sum_{j=1}^{n-1} \hat{\gamma}_j u_j(x_j) + u_n(x_n)}{\sum_{j=1}^{n-1} \hat{\gamma}_j [1 - u_j(x_j)] + 1 - u_n(x_n)}, \quad (23)$$

Denote $U = U(x)$ and $u_j = u_j(x_j)$ and let $\hat{h}_1, \dots, \hat{h}_{n-1}$ be the estimators of h_1, \dots, h_{n-1} derived from (18). We have $h_j = \ln(\gamma_j)$ so that $\gamma_j = e^{h_j}$. Thus the γ_j in expressions (19) and (20) can be estimated by $\hat{\gamma}_j = e^{\hat{h}_j}$. We have, from a Taylor's expansion of the function $e^{\hat{h}_j}$ at h_j , $\hat{\gamma}_j \approx e^{h_j} + e^{h_j} f_j = \gamma_j (1 + f_j)$, where $f_j = \hat{h}_j - h_j$. Substituting $\gamma_j (1 + f_j)$ for $\hat{\gamma}_j$ in (23), we obtain, from (21) and (22):

$$\ln\left(\frac{\hat{U}}{1 - \hat{U}}\right) \approx \ln\left(\frac{W_n^{-1}U + \sum_{j=1}^{n-1} \gamma_j u_j f_j}{W_n^{-1}(1 - U) + \sum_{j=1}^{n-1} \gamma_j (1 - u_j) f_j}\right) \quad (24)$$

The right hand side of (24) is of the form

$\ln\left(\frac{U + W_n \delta_1}{(I-U) + W_n \delta_2}\right)$ where:

$$\delta_1 = \sum_{j=1}^{n-1} \gamma_j u_j f_j \text{ and } \delta_2 = \sum_{j=1}^{n-1} \gamma_j (1 - u_j) f_j .$$

A Taylor's expansion in (δ_1, δ_2) at $(0, 0)$ to the linear term would yield

$$\begin{aligned} \ln\left(\frac{\hat{U}}{1-\hat{U}}\right) &\approx \\ \ln\left(\frac{U}{1-U}\right) + W_n U^{-1} \sum_{j=1}^{n-1} \gamma_j u_j f_j - W_n (1-U)^{-1} \sum_{j=1}^{n-1} \gamma_j (1-u_j) f_j \end{aligned} \quad (25)$$

On simplification, we have:

$$\ln\left(\frac{\hat{U}}{1-\hat{U}}\right) \approx \ln\left(\frac{U}{1-U}\right) + \frac{W_n}{U(1-U)} \sum_{j=1}^{n-1} \{\gamma_j (u_j - U)\} (\hat{h}_j - h_j) \quad (26)$$

Thus $\ln(\hat{U}/(1-\hat{U}))$ can be expressed as:

$$\ln\left(\frac{\hat{U}}{1-\hat{U}}\right) \approx \ln\left(\frac{U}{1-U}\right) + aA'Z ,$$

where

$$a = \frac{W_n}{U(1-U)} ,$$

A and Z are column vectors given by

$$A = (\gamma_1 [u_1(x_1) - U(x)], \dots, \gamma_{n-1} [u_{n-1}(x_{n-1}) - U(x)])'$$

$$Z = (\hat{h}_1 - h_1, \dots, \hat{h}_{n-1} - h_{n-1})' .$$

Hence we obtain:

$$\text{Var}\left(\ln\left(\frac{\hat{U}}{1-\hat{U}}\right)\right) \approx a^2 A' \text{Cov}(Z) A . \quad (27)$$

Here $\text{Cov}(Z)$ is simply the covariance matrix of the least squares estimator of $(h_1, \dots, h_{n-1})'$ and is

available from many standard statistical software. A confidence interval for $Q = \ln\left[\frac{U(x)}{1-U(x)}\right]$ can first be constructed as

$$\hat{Q} \pm z_{\alpha/2} \sqrt{\hat{a}^2 \hat{A}' \text{Cov}(Z) \hat{A}},$$

where \hat{Q} , \hat{a} , \hat{A} are obtained by replacing the unknown parameters in the expressions of respectively Q , a , A with the corresponding sample estimates, and $z_{\alpha/2}$ is the $100(1 - \alpha/2)$ percentile of the standard normal distribution. The lower and upper limits of this interval can then be transformed using the inverse transformation $\frac{e^Q}{1+e^Q}$ to obtain a confidence interval for $U(x)$.

5. Simulation study

In order to test the above formulation and its corresponding estimation procedure, three sets of simulated data involving 4 attributes were analyzed. The weights for these four attributes were fixed to .05, .1, .25 and .6 for all data sets. The first two values were intentionally chosen small and close to each other to find out if the proposed estimation procedure is capable of differentiating them. The weight of the last attribute was chosen to be large (.6) to test whether the proposed procedure in practice provide estimates beyond the limit. From these weights, the ratio W_j/W_4 ($j=1, \dots, 3$) were then calculated and using Excel program a set of random normal error with mean zero and standard deviation equal to .5 were added to the log of these ratios.

Table 1, provides a copy of the values used to simulate data under the first set of assessments which is based on the relative importance of various attributes to the most preferred one W_4 .

True values	True ratios W_j/W_n	$h_j = \ln(W_j/W_n)$	$f_j =$ normal errors	$h_j + f_j$	simulated ratios, r_j
0.05	0.0833	-2.4849	-0.0723	-2.5572	0.0775
0.10	0.1667	-1.7918	0.4645	-1.3273	0.2652
0.25	0.4167	-0.8755	-0.2065	-1.0820	0.3389
0.60					

Table 1. Simulated values of relative importance with respect to the most preferred attribute.

The above weights were also used to obtain the ratios W_j/W_1 ($j=2, \dots, 4$) and a set of random normal error terms with mean zero and standard deviation equal to .5 was then added to the log of these ratios. Table 2. provides a summary of the values used to simulate the second set of assessments which is based on the relative importance with respect to the least preferred attribute W_1 .

True values	True ratios W_j/W_1	$\gamma_j = \ln(W_j/W_1)$	$f_j = \text{normal error}$	$\gamma_j + f_j$	simulated ratios, r_j'
0.05					
0.10	2.000	0.693	0.0518	0.7449	2.1063
0.25	5.000	1.609	-0.1536	1.4559	4.2883
0.60	12.000	2.485	0.3370	2.8219	16.8084

Table 2. Simulated values of relative importance with respect to the least preferred attribute.

The above ratios (r_j and r_j') constituted our 1st simulated data set, which was then treated as the dependent variable in Eq. 18. Using the SPSS regression procedure we estimated the regression coefficients h_j for this data. The results are given in table 3.

	Unstandardized Coefficients		Standardized Coefficients	t	Sig.
	B	Std. Error	Beta		
$h1$	-2.561	.144	-1.142	-17.732	.000
$h2$	-1.572	.191	-.496	-8.226	.004
$h3$	-1.094	.191	-.345	-5.724	.011

Table 3. SPSS output for the 1st simulated data set

From Table 3 we note that all coefficients are significantly different from zero (p-value less than .05). Using these estimates, from (13) and (11) we have calculated the weights of all attributes, which are reported in table 4. These weights may be considered as estimates of the true weights, which are also given in this table.

We note that the estimates are very close to their corresponding true values and the estimated weight for W_1 , although quite small (.0477), is statistically significant and different from zero.

True weights W_j	Estimated weights	
	1 st data set	3 rd data set
0.0500	0.0477	0.0553
0.1000	0.1282	0.1219
0.2500	0.2067	0.1965
0.6000	0.6174	0.6263

Table 4. The true weights used and their corresponding estimates obtained from the analysis of the first data set.

To generate the second data set we repeated this simulation procedure using the same weights and error terms but, we added two constants, $c_1 = -.7$ and $c_2 = .7$ to represent bias in measurement. These values of bias, in practice, may be interpreted as assessing, on average, the relative weights of the attributes with respect to the best attribute half of the actual values and assessing the relative weights of the attributes with respect to the least preferred one, about twice the actual values. Finally, a third data set similar to the second one was generated. However, for this data set, the value of c_1 and c_2 were set to $.7$ and $.5$ respectively which are close to each other. These simulated data sets along with their corresponding true values are reported in Table 5. We note from this table that the simulated values are quite different from their corresponding true values.

True ratios	2nd data set	3rd data set
0.08	0.038	.156
0.17	0.132	.534
0.42	0.168	.683
2	4.242	3.473
5	8.635	7.070
12	33.848	27.712

Table 5. Simulated data sets

For analysis of these data sets, we added two indicator variables to matrix D , as discussed in section 3.2, to account for possible bias in assessment. The estimated regression coefficients obtained from SPSS for the second data set is given in Table 6. As we note from this table, the estimates of ratios (h_j), although close to previous values, are not statistically significant (p -values greater than $.05$). This is due to addition of two more columns to matrix D that caused the degree of freedom of residual in the regression model to be reduced from 3 to 1.

	Unstandardized Coefficients		Standardized Coefficients	t	Sig.
	B	Std. Error	Beta		
c_1	-.183	.332	-.052	-.551	.679
c_2	.435	.332	.124	1.309	.415
h_1	-3.087	.407	-1.013	-7.591	.083
h_2	-1.959	.311	-.455	-6.308	.100
h_3	-1.483	.311	-.344	-4.776	.131

Table 6. Regression output for the 2nd simulated data

Similar to the second dataset, the third simulated data was also analyzed using the SPSS program. The estimated regression coefficients for h_j obtained from the regression model were identical to those reported for the second dataset. However, the estimates of c_1 and c_2 , reflecting bias in assessments,

were 1.217 and .248 respectively, which are different from the previous case as expected. These results may be considered as an indication of the stability of the estimation procedure.

Finally, we have reanalyzed the third data set by only adding a column of 1 to the above matrix D . That is, we assumed equal bias for the data obtained under the two assessment methods and introduced a unique intercept. The regression coefficients obtained from analysis of this data set is given in Table 7, and the estimated weights derived from these regression coefficients are reported in Table 4, which may be compared with their corresponding true values.

	Unstandardized coefficients		Standardized coefficients	t	Sig.
	B	Std. Error	Beta		
Constant	.732	.230		3.190	.086
h_1	-2.428	.187	-1.040	-12.954	.006
h_2	-1.637	.338	-.444	-4.846	.040
h_3	-1.159	.338	-.314	-3.431	.075

Table 7. Estimated regression coefficients for the third data set

From Table 7, we note that the estimated regression coefficients are all close to their corresponding true values reported in Table 1. and are all statistically significant at $\alpha = .1$. This is due to the fact that, in generating this data set the values of c_1 and c_2 were chosen to be close to each other. Thus, the assumption of equal intercept for equations (19) and (20) is a plausible assumption. This resulted in gaining an additional degree of freedom for the error terms (residuals). The estimated weights are reported in Table 4. As we note from this table, the estimates are very close to those obtained from the first data set and to their true values. It is also interesting to note that the estimate of a common bias for the two assessment procedures is .732, which is close to the actual values used for generating this data set but a bit inflated.

To examine the effect of the proposed estimation procedure on assessment of the overall utility U , we have used our estimates for the third dataset to assess the overall utility of two hypothetical scenarios (outcomes) with fixed utilities $\mathbf{u}_1 = (.25, .5, .75, 1)$ and $\mathbf{u}_2 = (1, .75, .5, .25)$. These scenarios were intentionally chosen with increasing and decreasing utilities so that we can examine the effect of weights on both small as well as large utility values.

To compare the estimated utilities of these scenarios with those obtained from the raw data (not treated for response error), we first derived two sets of weights corresponding to the two assessment procedures for the third simulated data set given in Table 5. Then, using (3) we calculated the multi-attribute utilities of these scenarios based on the two assessment methods as well as their estimates using a 95% confidence interval. These weights along with their corresponding utility values are given in Table 8.

As we note from Table 8, the estimated utilities are very close to their corresponding true values whereas, the utilities calculated on the observed (untreated) weights are far from their corresponding true values. We also note that, the 95% confidence intervals for the utilities of these scenarios incorporate the corresponding true values.

	True weights	Weight based on 1st method	Weight based on 2nd method	Estimated weights
w_1	0.05	0.0657	0.0255	0.0553
w_2	0.10	0.2250	0.0885	0.1219
w_3	0.25	0.2878	0.1801	0.1965
w_4	0.60	0.4214	0.7059	0.6263
U_1	0.85	0.8916	0.7662	0.8485 ± 0.0398
U_2	0.40	0.3584	0.4838	0.4015 ± 0.0037

Table 8. Estimates of weights and utilities for the third data set

6. Conclusions

Assessment of parameters of multi-attribute utility functions is subject to measurement error. We took a statistical approach to deal with error of measurement and in the context of multi-attribute additive model provided an estimation procedure for assessing the relative weights. The estimation method is based on assessing ratios of weights and for n attributes, requires only $2(n-1)$ assessments. Consequently, we proposed two versions of the swing assessment method, which provide directly the required ratios.

A major advantage of the estimation procedure is that due to its unique formulation it keeps all the parameters within the required boundaries without imposing any constraint. That is, the estimates of weights will be always between zero and one and always satisfy Eq. (1).

Since the estimation method is based on regression techniques, one can easily test a variety of hypothesis that may have significant importance in application using the standard inference methods available for linear models. For example, we can test significance of individual weights or hypothesize and test whether the relative weights of a set of attributes are equal.

Another unique and important contribution of the proposed method is providing confidence interval for the overall utility function. Availability of such a confidence interval will provide an opportunity for DSS developers to accommodate measurement error in analysis of decision problems and develop systems that compare, sort or group multi-attribute alternatives based on their "true" utilities.

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