A Procedure for Multiattribute Reverse Auctions with two Strategic Parameters

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Abstract

Reverse auctions have been used in the procurement of goods and services. Single-attribute, typically price-only, auctions are well known and have been widely used. Multi-attribute auction mechanisms have also been proposed and implemented. The dilemma that designers of multi-attribute auction mechanisms face is the trade-off between revelation of the buyer (auction owner) and the ability of the sellers (bidders) to construct effective offers in any given round. These are the offers which are preferred by the buyer over offers that were made in the earlier round. Full transparency requires that the buyer gives complete preference information to the bidders; it is often rejected by the buyers because of the future implications when competitors obtain this information. Zero transparency renders auctions ineffective because bidders lack the information necessary for bid construction. This paper develops a procedure which transposed preference information into information about reservation levels which the bidders need to obey and which is updated after each round. The reservation level information is perturbed so that the bidders cannot compute the buyer's preferences. The perturbation is controlled by two parameters which have strategic character for the auction mechanism.

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Acknowledgments: The publication of the InterNeg Research Papers has been supported by the Natural Sciences and Engineering Research Council, the Social Sciences and Humanities Research Council, and the J. Molson School of Business, Concordia University.

1. Introduction

E-procurement is a key area of e-business and supply chain management in which catalogs and reverse auctions have been widely used (Anderson and Frohlich 2001; Jap 2003). On average, about 70% of corporate revenue is spent on purchasing; savings of 5% translate into hundreds of millions of dollars (Peleg 2003; Wagner and Schwab 2004). There are two kinds of auctions: single-shot and iterative. Iterative auctions, which allow bidders to revise their bids, are becoming prevalent in procurement (Parkes and Kalagnanam 2005). Consequently, this paper discusses the design of iterative auctions. One of the limitations of auctions is that they use a single attribute (i.e., price), which leads to inefficient agreements (Strecker and Seifert 2004) and is non-practical in many business transactions (Teich, Wallenius et al. 2004).

Procurement of more complex goods and services often requires consideration of multiple attributes (e.g., total costs of ownership components, quality, risk and schedules). One of the popular types of auctions used in procurement is the iterative English reverse auction in which sellers submit bids in order to sell an item to a single buyer. Two types of iterative auctions are possible: synchronous and asynchronous. An auction is synchronous if every seller makes at most one bid in each round. The buyer selects the best bid as the reference bid and presents it to the sellers. The sellers use the reference bid to construct their bids in the next round bid. In addition, the buyer may provide other information to help sellers construct their bids. Asynchronous auctions allow sellers to bid at any time until the auction's deadline. The best bid is shown to the bidders. If a better bid is submitted then it replaces the previous best bid. In this paper we focus on synchronous auctions; however, the procedure proposed in this paper can be adapted to the asynchronous auctions.

Auction design has traditionally focused on the construction of rules which govern the behavior of auction participants so that their auctions lead to a "desired" market outcome. The outcome is the final allocation of the goods and money. The "desired" aspect of the outcome is the auction initiator (the buyer in our case), profit or revenue maximization, or it is the creation of an efficient market (Kittsteiner and Ockenfels 2006). The rules specify the winner determination formula, auction duration and the type of deadline (extendible or fixed), types of bids (sealed or open), and so on.

In synchronous iterative single-attribute auctions the rules determine whether all bids are open and posted so that they are visible to all bidders, only some bids are open and visible, or only the best bid made in a given round is open and visible. Either of these options is sufficient for the bidders to decide on bidding in the next round. Therefore, the rule defining acceptable bid is simple—every submitted bid must exceed the last posted bid. This rule assures that the time-order of bids is the same as the profit-order for the buyer, that is, later bids are better than earlier. The concept "better than" is easily operationalized by the explicit auction criterion, which is the single attribute.

Multiattribute auctions cannot have such a rule because there is no auction criterion that is explicit and known to all participants. Ways to overcome the lack of an explicit criterion include: (1) pre-selection of bidders so that only bidders who are known to meet the additional criteria are included; (2) giving incumbents an advantage because their qualifications are known; and (3) the use of disclaimers such as "the lowest bid may not be awarded the contract" (Bichler and Kalagnanam 2005; Engelbrecht-Wiggans, Haruvy et al. 2007; Schoenherr and Mabert 2007). In these types of auctions either the selection or bidding process are modified so that a single-attribute auction can be used.

The results of such auction modification are mixed because of collusion and selection of inferior offers (Elmaghraby 2004; Katok and Wambach 2011). In some situations the process becomes an auction in name only, as is the case with an auction in which neither the winner nor any other participant is awarded the contract.

Another, seemingly simple approach is to give bidders all information which the buyer uses in order to analyze and compare bids. This somewhat complicates the computation because the bidders need to optimize using both their own and the buyer's information (e.g., utility or a scoring function). It may also induce the buyer to engage in strategic misrepresentation and announce a utility function with the aim of pushing the sellers to make favorable bids (Burmeister, Ihde et al. 2002).

This approach is unacceptable when buyers do not want to disclose their preferences for strategic, competitive, or other reasons (Burmeister, Ihde et al. 2002; Parkes and Kalagnanam 2005). In the context of multiattribute bidding, this means that the bidders do not know how to bid; they cannot make tradeoffs that take the buyer's preferences into account and they may misinterpret the buyer's preferential directions. The bidders may make strong assumptions about the buyer's utility and bid accordingly. This may be acceptable if their knowledge of the buyer's preferences is accurate and the buyer accepts an inefficient winning bid.

Another option has been proposed by economists. This option rests on the assumption that all attributes can be expressed in monetary terms so that only two items need to be considered: (1) price, and (2) monetized attributes, which typically represent costs—for the sellers and value (income)—for the buyer. When an assumption is added that these two terms are monotonic and the buyer compares bids using the difference between value and price, then the sellers can determine the buyer's preferential order of the alternatives.

The attribute monetization methods have been widely implemented and tested (e.g., Che 1993; Strecker and Seifert 2004; Bichler and Kalagnanam 2005), and they are considered a standard in the auction literature (Parkes and Kalagnanam 2005). These methods, based on two-attribute monetary value functions; are appealing because they allow buyers and sellers to integrate and trade off all attributes included in the cost function (Strecker and Seifert 2004). On one hand, the bidder may choose a bid among his/her indifferent alternatives (i.e., different bids which yield the same utility for this bidder) that yields the highest utility to the buyer; on the other hand, the owner evaluates bids based on the total utility of bids and chooses the highest one. The limitation of this method is the underlying assumption that all attributes can be measured with money. The assumption is questionable, if one considers such attributes as trust, commitment, or color.

The design of auction mechanisms that rely on attribute monetization involves the construction of rules that help the sellers to make "progressive" bids; i.e., bids which are better for the buyer than the bids made earlier. The information conveyed to the sellers is about the buyer's preferences and it is either complete or incomplete but sufficient to assure the auction convergence. A different approach has been proposed by Teich, Wallenius et al. (1999) in which the sellers are informed about a path in the space of alternatives.

In this paper we build on the approach proposed by Teich, Wallenius et al. (1999) in the sense that any information conveyed to the sellers refers to the alternative space. Furthermore, rather than inform the sellers about preferential direction(s) and/or alternatives preferred over any given alternative, we restrict information to the acceptable alternatives so that no preferential information needs to be conveyed.

Given or focus, we are not concerned here with the complete set of auction design rules which assures that given outcomes are met. Instead, we are concerned with rules which assure that no alternative is removed which could yield the desired outcome. Whether such an outcome is achieved depends on the sellers' behavior, which is not discussed here.

The proposed procedure is controlled by two strategic parameters. These parameters determine the acceptable margin of error of accepting a bid which is inefficient. Following Bellosta et al. (2008, p. 402) we say that the winning bid is *efficient* when it is a feasible alternative and no seller, except possibly the winning seller, can provide a bid that is better than the winning bid. Strictly speaking, an efficient alternative takes into consideration the buyer's as well as the bidders' utility functions: "no better bit" means that there is no bid that would yield higher utility for at least one from the pair (buyer, seller) and not worse for any of them. Because we do not consider here bidders' preferences and utilities, we use the term "efficient alternative" to describe a bid which maximizes the buyer's utility over all possible bids.

By controlling the value of the strategic parameters the buyer controls the ability of the bidders to determine her preferences and the possibility of the winning bid to be efficient. A relationship is roughly that the less likely for the bidders to know the buyer's preferences yet be able to bid effectively, the more likely is that the winning bid is ninefficient.

The motivation for the proposed procedure derives from behavioral experiments in which we compared multiattribute auctions and multibilateral negotiations (Yu, Kersten et al. 2008) and the requirement to adapt the mechanisms used in these experiment to more realistic settings. We found that the auction mechanism based on the price/costs function is inadequate to such problems as procurement of logistic services (Pontrandolfo, Wu et al. 2010) and energy trading (Block, Collins et al. 2010).

2. Multiattribute reverse auctions

The two key tasks in multiattribute auctions are: (1) representation buyer of the buyer's preferences so that there are some means to compare bids; and (2) specification of the feedback information which the sellers need to receive in order to construct bids.

2.1 Preference representation

There two main types of preference representation methods (Fishburn 1976; Dieckmann, Dippold et al. 2009):

1. Compensatory methods, which include additive value functions and more complex utility functions based on multiattribute utility theory (MAUT); and
2. Non-compensatory methods, which include attribute lexicographic ordering and the Tchebychev measure.

Compensatory methods are based on the assumption that decision makers' preferences are defined on both attributes and attribute values, and that they can formulate trade-offs between attributes and between attribute values. This assumption allows for the aggregation of preferences into some kind of function which measures the worth of an alternative. The measure is a utility function[1]. Multi-attribute auction mechanisms have been designed using scoring functions or utility (e.g., Bichler 2001; Beil and Wein 2003; Engel and Wellman 2010). Additive linear functions (i.e., weighted sum) have been implemented in e-sourcing systems offered by

[1] We use the term "utility" loosely so that it covers utility functions, value functions and simple rankings.

B2emarkets.com (now bravosolutions.com) and Perfect Commerce (perfect.com), (both current December 2010).

A specific class of compensatory methods is total costing in which all attributes and their values are transformed into monetary values. These methods can be used only when the attributes can be priced; examples include A.T. Kearney Procurement & Analytic Solutions (ebreviate.com) and CapGemini IBX (ibxeurope.com), (both current December 2010).

Non-compensatory models have been proposed to evaluate bids at the attribute level but not between the attributes. The two well-known methods are: lexicographic ordering and the Tchebychev function. Lexicographic approaches are simple heuristics in which the attributes are ordered from the most important to the least important. The alternatives are compared first using the most important attribute. If they do not differ on this attribute, then the second most important is used, and so on. These heuristics were found to perform well and sometimes better than a compensatory method (i.e., conjoint analysis) in *ranking* of alternatives but not in *rating* them (Dieckmann, Dippold et al. 2009). In Dieckmann, Dippold, et al. study as well as earlier studies (e.g.., 82–96) the participants were given little time and need to choose from among several alternatives. When participants were given more time and could explore information, compensatory models outperformed lexicographic strategies. If the number of alternatives is large, then lexicographic models may fail because they do not allow for large difference in values of several attributes of lower importance to outweigh a small difference in the value of a more important attribute.

Bellosta et al. (2004) proposed the use of Tchebychev distance to represent the buyer's preferences. The non-compensatory character of this distance allowed the authors to suggest feedback based on attribute values. The sellers need not consider an tradeoffs, instead their bids have to contain a value greater than the previous best bid on at least one attribute and not worse on any attribute.

Bellosta, Kornman and Vanderpooten (2008) proposed MERA, a procedure for mechanism design for both synchronous and asynchronous multiattribute reverse auctions in which, in addition to Tchebychev distance, lexicographic ordering and weighted sum function can be incorporated. The framework relies on the notion of reservation levels for which constructing the preference aggregation method is used. In this paper we also use reservation levels for action design. The key difference between our proposal and MERA is in the space in which these levels are constructed. While in both procedures the levels originate in the utility space, we transform the reservation from the utility space to the space of alternatives. This has an important and desirable impact on the information feedback discussed in the next section.

2.2 Feedback information

A common concern in multi-attribute auctions pertains to the exchange of additional information relevant to the buyer's preferences.

Several rules have been designed with different feedback information provided to bidders during auctions, including: complete value function, winning bids (with/without value), and all bids (with/without ranking).

In the framework proposed by Bellosta et al. (2008) the information passed by the owner depends on the way she constructs her representation. When the representation includes a linear additive utility function, then the owner passes this utility and its lower bound. When the preferences are represented as a lexicographic aggregation model or a Tchebychev function, then the owner passes bounds imposed on the attribute values. This dependency is difficult to reconcile with the requirement that the owner does not make her preference model public (Burmeister, Ihde et al. 2002; Parkes and Kalagnanam 2005). The owner's inability to keep her preference private may force her to use a different model, which she does not know, agree with, or is inappropriate to the particularities of the problem.

Teich, Wallenius et al. (1999) suggest a feedback rule in which the buyer prescribes a preference path, an ordered set of combinations of prices and non-priced attributes. The preference path begins with an anchor point and the rule specifies that a point further from the anchor is preferred by the owner over the point that is closer to it. This allows the sellers to decrease the worth of their bids (as seen by the buyer) by proposing a combination that is more preferred by the buyer than that combination previously proposed. Burmeister, Ihde at al. (2002) note one drawback of this method which is bidders' restriction in their choices, i.e., they are only allowed to bid on the preference path. Another limitation is the possibility for sellers to use the preference path to construct the buyer's utility function.

The feedback rule described by Bellosta et al. (2008) depends on the preference representation method. For a compensatory method the feedback is the buyer's preference aggregation function. The feedback also includes the minimum scoring value which is the value of the best bid made in the earlier round plus an arbitrary increment. This feedback allows the sellers to determine if they are willing to make bids which scoring value exceeds the minimum. Informing the sellers about the buyer's preferences is the primary limitation this approach. As we have mentioned above, in many instances buyers' are not willing to provide this information. Therefore, we propose a procedure in which the buyer's preferences are not revealed yet the sellers obtain information which allows them to make progressive bids.

3. Problem representation

3.1 Preliminaries

The proposed procedure for multiattribute auction has two types of components: (1) the auction owner component, and (2) the bidder component. There are one or more bidders and they may behave according to the same or a different set of rules. We consider here reverse auctions, therefore the owner is the buyer and bidders are sellers. The procedure may, however, be modified to standard auctions in which the owner is the seller and bidders are buyers.

Reverse auction A is a set of n-tuples (collections); A counts elements which describe the owner's (buyer) and all J bidders' (sellers') problem representations. Sellers make bids in each round t, t=1 ,…, T):

$$A = \{P_t, I_t, B_{jt}, O_{jt}\},\ (\mathrm{t} = 1, \dots, T;\ j = 1, \dots, J) \qquad (1)$$

where, for round t (t = 1, …, T):

1. The auction's owner (buyer) problem in representation P_t;
2. Information (feedback) which the auction owner presents to the bidders I_t;
3. Representation of individual bidders' decision problems B_{jt}, (j = 1, ..., J); and
4. Bids, which are solutions that the bidders construct by solving problems and submit, O_j, (j = 1, ..., J).

Let us observe that, for example, if in a round t (index t is omitted now to keep clear presentation) for J = 3 the tuple $A = \{P, I, B_1, O_1, B_2, O_2, B_3, O_3\}$ represents ability (I) of owner to communicate the problem to three bidders and their reactions to the information to this message. Problem representations B_1, B_2 and B_3 of bidders are unknown to the owner. Resultant reactions O_1, O_2 and O_3 are known to them. In the sequel we assume elicited additive utility for the owner. The bidders' representation remains uncovered in the analysis.

It is important to allow for the separation of these representations. In particular, two types of separation are required:

1. Owner framing: while I is derived from P, the same information I may be obtained for different representations P, and vice versa; and
2. Bidder separation: irrespectively of the representation B_j bidder j is using, she formulates her bid O_j (j = 1, ..., J), in a manner required by the auction protocol.

The two separations are conceptually similar in that they both state that the information passed by one person (entity) to another does not depend on the way this person formulates and solves her decision problem. We are concerned here with the perspective of the buyer and information that the buyer presents to, and obtains from, the sellers. The sellers are completely independent—they may construct any type of their own problem representation and they may make any offer they wish, providing they do it according to the auction protocol.

3.2 Representation of the buyer's decision problem

Buyer's problem representation comprises a set of feasible and acceptable alternatives X and the buyer's utility function U, that is, $P = \{X, u\}$. We consider alternatives which are described using N attributes[2] ($N \geq 1$), each having J values, hence set X is assumed to be discrete. It has $L = |X| = \prod_{n=1}^{N} J_n$ alternatives and is bounded[3].

Let us introduce the following notation:

x_j^n – j-th value of attribute n, (j = 1, …, J_n; n = 1, ..., N), $X_n = \{x_j^n, j = 1, \dots, J_n\}$;) (2)

$\boldsymbol{x}_l = [x_{jl}^n]$ – l-th alternative, l = 1, ..., L;

X – set of feasible alternatives, $X = \{\boldsymbol{x}_l = [x_{jl}^n], l = 1, \dots, L\}$.

Attributes may be discrete or continuous variables. If they are discrete (e.g., categorical or nominal), then their values are known to auction participants. If they represent a continuous variable (e.g., distance or weight), then we assume that only a discrete subset is considered. The permissible attribute values are those which differ by no more than ε^n, which is the smallest meaningful increment of attribute x^n (e.g., a centimeter, gram or dollar).

In order to describe an alternative's evaluation, we assume, without loss of generality, that the attributes are meaningful to the owner in the alternatives' comparison – we assume that the owner wants all attributes be achieved at the highest possible levels[4].

[2] Attributes serve not only to describe alternatives but they also play important role in their evaluation.

[3] The discretization of X is not necessary but it simplifies the process. It also has little practical implications because in most, if not all, auctions there are the smallest allowable units, be it dollars or cents, meters or grams.

[4] Qualitative attributes (e.g., color or mark) have no natural order (e.g., one cannot say what is the increment from attribute value "blue" to value "green". Such attributes can be ordered with a subjective scale, e.g., according to their utility. This means that the auction participants may have very different orderings of a qualitative attribute. The buyer may provide her preferential order or exclude some values of these attributes throughout the auction. To be consistent with our perspective that buyers do not want

Consequently, we assume that there is the owner's numerical evaluation $u : X \rightarrow R$ of alternatives from the set X. We also assume that every bid, which can be accepted, is an element of set X. This means that bids O_j introduced in (1) are feasible alternatives $\boldsymbol{x}_l$. The utility of the j-th bidder's offer $\boldsymbol{x}_j$ aggregates utility attribute and next it takes into account values of attributes.

For each attribute n, n = 1, ..., N, let us denote by w^n the weight representing the owner's preference for this attribute. Partial utility $u^n(x_j^n)$ describes the owner's preference regarding j-th value x_j^n (j = 1, ..., J_n) of the n-th attribute. It is a product of the attribute and attribute value weights, i.e.:

$$u^n(x_j^n) = w^n \, v(x_j^n).$$

The utility of an alternative $\boldsymbol{x}$ is the sum of attribute and attribute value weights. We assume that the utility for bids function is additive and monotonic (it may be nonlinear), i.e.

$$u_l = u_l(\boldsymbol{x}) = \sum_{n=1}^{N} w^n \, v^n(x_j^n). \tag{3}$$

Useful but not necessary information is about an alternative that yields the highest utility value for the buyer. We call this ideal alternative $\hat{\boldsymbol{x}} \in X$. This means that there is no feasible alternative which yields higher utility.

3.3 Reservation levels

Auctions literature suggests specification of reservation levels (Milgrom and Weber 1982; Walley and Fortin 2005). These are the bounds used to distinguish acceptable attribute values from unacceptable ones. The acceptability of an alternative depends on its feasibility and utility. A given utility value is used as a threshold so that alternatives which yield lower utilities are deemed unacceptable and those that meet or exceed the threshold are *acceptable*.

In single-attribute auctions, bidders are informed about the reservation level (reserve price). Bids that fell below that value are rejected. Because the first bid has to be higher than the reservation level and a subsequent bid has to be higher than the bid it precedes, the initial reservation level need not be revised. Bids play a role of a reservation levels; there are as many changes in the de facto reservation levels as there are bids.

In multiattribute auctions we need a different approach.

In the proposed procedure, in one round, sellers submit bids not knowing about their order. That is, they do not know which bid is the winning bid and whether the buyer prefers their bid over other bids. Before they move to the next round they may be informed about the winning bid. Our procedure allows for this option. The announcement is, however, not sufficient for bidders to be able to propose bids that yield higher owner's utility value than the utility of the winning bid.

Before moving to the next round, the bidders need to obtain information about the acceptable bids. This information is contained in the announced (by auction owner) revised reservation levels. The reservation levels thus play a similar role in the proposed procedure as in a single-attribute auction, but they are revised after each round. The purpose of the revisions is to guide the sellers into the subset of X which is acceptable to the owner.

For example, in a single-attribute auction the best bid in round t is \$512. This informs the

to disclose their preferences, we choose the latter option.

bidders that in round t+1 any bid below $512 is unacceptable and any bid above $512 is acceptable. If there are two attributes—price and delivery time—then information that the best bid in round t was $406 and 35 days does not provide the bidders with sufficient information. If, however, the owner announces that either the price has to be higher than $410 and delivery time 33 days, or that the price has to be higher than $390 and delivery 29 days, then the bidders can make acceptable bids.

Assuming that the auction may be represented as a series of bidding rounds t (t = 1, 2, ..., T), reservation levels are determined at the beginning of rounds t, (t = 0, 1, ...T-1). The process of reservation-level revision resembles a single-attribute auction with the difference being that here the revised values need to be computed. In a single-attribute auction the most recent bid value becomes de facto new reservation level and the next bid cannot be below this value. In the multiattribute case, the levels need to be re-evaluated so that they reflect the buyer's preferences over multiple attributes.

Another difference between single- and multi-attribute auctions is that, in the former case, a single reservation level is sufficient to restrict biddings to all alternatives that the buyer prefers over a particular reservation levels. This is not the case in multiattribute auctions. The preference is determined by the minimum acceptable utility value which then needs to be transformed into reservation levels set for all attributes.

We say, that, the set X_r is acceptable with the reservation level r, $r \in \mathbf{R}$, for the owner when all its elements meet the *reservation level condition*, i.e.:

$$X_r = \{\boldsymbol{x} \in X : u(\boldsymbol{x}) \geq r\} = u^{-1}([r, +\infty[), \tag{4}$$

where r represents the minimum acceptable utility value.

The reservation level r means that every bid which yields lower utility than r is rejected; it is considered to be an infeasible bid. To stress this characteristic, we call X_r *r-feasible* set.

We assume that the buyer does not want to inform the bidders about the minimum acceptable utility value because this would be tantamount to informing them about her utility function. Therefore, she has to transform the above condition so that it is defined on the attribute rather than utility values. That is, the buyer has to transform utility reservation level r to attribute reservation levels which she can pass to the bidders. We assumed (see Section 3.2) that the owner wants all attributes be achieved at the highest possible levels. Therefore, the attribute reservation levels are lower bounds.

Note that some attributes may be nominal. For these attributes lower bounds are not meaningful. These attributes' values are ordered by the buyer's preferences which are unknown to the bidders. Therefore, we assume that the bound, which is defined for nominal attributes, divides their values into acceptable and unacceptable ones. For example, consider color as an attribute with four feasible values (black, grey, red and white). A bound dividing the feasible set may set black and grey as unacceptable and red and white as acceptable. The bidders are told that only bids in which color is either red or white can be accepted.

To illustrate the construction of the lower bounds based on (4) consider an example shown in Figure 1a. Set X of feasible alternatives consists of all points shown in the Figure 1. Set X_r^A is the set of all points on and above line r.

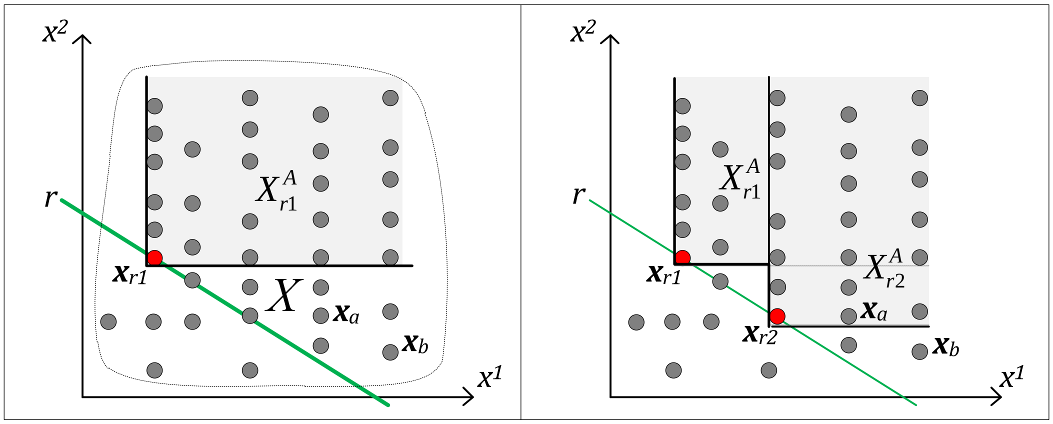

Figure 1. (a) Single-point reservation levels; (b) Two-point reservation levels.

Let's assume that the reservation level r is determined by alternative $\boldsymbol{x}_r$, that is, $u(\boldsymbol{x}_r) = r$. Alterative $\boldsymbol{x}_r$ is said to be the *reference alternative* because it is used to construct *r-acceptable* set X_r^A, as follows:

$$X_r^A = \{\boldsymbol{x} \in X: [x^n] \sqsupseteq [x_r^n]\}, \quad (5)$$

where $\sqsupseteq$ denotes: (i) the relation ≥ if x^n is a numerical variable, or (ii) it is another preferential order that divides set X_n of attribute values into acceptable and unacceptable subsets.

Taking into account monotonicity of the owner's utility, we obtain that

$$X_r^A \subset X_r. \quad (6)$$

This means that X_r^A comprises alternatives with utility values being not lower than r. Note that information about X_r^A can be easily conveyed to the bidders. Information about X_r^A is included in I defined in (1). For numerical attributes it is sufficient to include in I the requirement that the attribute value cannot be lower than x^n. If the attributes are nominal, then I must include either all acceptable attribute values or all unacceptable values.

Information included in I, which describes X_r^A, is obtained by transformation of the reservation level condition given in (4) from the utility space to the alternative space. To indicate that it is obtained by the process of selection of utility value r and reference alternative $\boldsymbol{x}_r$, we call this information *r-reservation* levels.

In what follows, we assume, for simplicity, that alternative values are numerical. In this case X_r^A comprises points inside the rectangle ($\boldsymbol{x}_r \oplus \mathbf{R}_+^{\mathbf{N}}$).

The use of utility value r as the reservation level causes that feasible set X to be split into r-feasible sets X_r and $X - X_r$. Consequently, the use of r-reservation levels partitions set X_r into set of r-acceptable alternatives X_r^A and $X_r - X_r^A$.

3.4 Design parameters

The construction of set X_r^A defined in (5) may result in a loss of r-feasible alternatives (i.e., in the case where $X_r - X_r^A$ is non-empty). This means that some r-feasible alternatives are not r-acceptable; their utility value is greater than r but they do not exceed one of the attribute reservation levels. This is the consequence of the way the attribute reservation levels are specified rather than the intention of the buyer in placing a restriction on bids.

This situation is illustrated in Figure 1(a). The buyer desires that the bids' utility exceeds utility

r,. However by selecting only one point ($\boldsymbol{x}_{r1}$) to decide on the r-reservation levels, she removes several acceptable alternatives where utility is not lower than r. For example, the utility of both $\boldsymbol{x}_a$ and $\boldsymbol{x}_b$ is higher than r.

If $\boldsymbol{x}_r$ is a bid, then the solutions of inequality $u(\boldsymbol{x}) \geq r$ define the set X_r^A in (4), which is not empty. In the case where X is a vector-space structure $\mathbf{R}^{\mathrm{N}}$, we may construct the following hyperplane:

$$H_{\mathrm{r}} = \left\{\boldsymbol{x} \in \mathbf{R}^{\mathbf{N}}: r = u(\boldsymbol{x}) = \sum_{\mathrm{n}=1}^{\mathrm{N}} w^n \ v^n(\mathrm{x}^{\mathrm{n}})\right\}.$$

Observe that $\bar{X}^{\mathrm{r}}=U^{-1}(r)=X \cap H_r$. Not all elements of $\bar{X}^r$ have to be included in the set which the owner presents to the bidders in order to receive bids with utility value not lower than r (see e.g. Figure 1(a)) and without revealing her utility. We show that if there are r-feasible alternatives which are not r-acceptable, then they can be used to expand the set of r-acceptable bids.

<u>Proposition</u>. The owner can operationally split the set of r-acceptable alternatives into alternatives which attribute values exceed attribute thresholds defined by a bid with utility r, and the remaining alternatives.

In the case of a finite set the split can be done by enumeration of bids and direct comparison of their attribute using relation $\sqsupseteq$ (5). If this relation is represented by the relation ≥ then the condition for a split can be phrased in terms of scalar products of vectors.

3.4.1 Construction of *r*-acceptable set

The fact that some r-feasible alternatives are not included in r-acceptable set X_r^A may lead to an auction which terminates with an inferior, Pareto-dominated, bid. This is the case when the bidders want to bid only on the excluded alternatives but are unable to do it. Consider, for example, the situation illustrated in Figure 1(a). If no bidder submits a bid that meets r-reservation levels (is an element of X_r^A), then an earlier bid with utility lower than r would be accepted. It is possible, however, that bidders would propose $\boldsymbol{x}_a$, $\boldsymbol{x}_b$ or some other alternative which is r-feasible but not r-acceptable.

We can increase the number of r-feasible alternatives by increasing the number of alternatives which are used to generate an r-acceptable set. In order to do this we increase the number of reference alternatives.

The case of two reference alternatives is illustrated in Figure 1(b). The introduction of the second reference alternative expands the r-acceptable set. That is:

$$X_{\{r1,r2\}}^A = X_{r1}^A \cup X_{r2}^{At} = \{\boldsymbol{x} \in X: \boldsymbol{x} \geqq \boldsymbol{x}_{r1}\} \cup \{\boldsymbol{x} \in X: \boldsymbol{x} \geqq \boldsymbol{x}_{r2}\}^5. \tag{7}$$

Note that while the utility of one of the reference alternatives is r, the utility of other reference alternatives may be greater than r.

Let $X^{\geqq}(\boldsymbol{x}_0)$ be a set comprising of feasible alternatives such that each attribute value of its elements is no smaller that corresponding attribute value of $\boldsymbol{x}_0$. That is:

$$X^{\geqq}(\boldsymbol{x}_0) = \{\boldsymbol{x} \in X: \boldsymbol{x} \geqq \boldsymbol{x}_0\}.$$

In (7) two reference alternatives are used to expand the r-acceptable set. In general, D such alternatives may be used. Hence we obtain:

$$X_{rD}^A = \cup_{d=1}^D X_d^A = \cup_{d=1}^D \{\boldsymbol{x} \in X: \boldsymbol{x} \geqq \boldsymbol{x}_{rd}\}\,, \tag{8}$$

[5] "≧" denotes that respective inequality holds for each attribute of an alternative.

where: alternative $\boldsymbol{x}_{rd}$, is d-th reference alternative (d = 1, …, D), such that $U(\boldsymbol{x}_{rd}) \geq r$, and there is at least one reference alternative $\boldsymbol{x}_{rd^*}$ for which $U(\boldsymbol{x}_{rd^*}) = r$. Reference alternative $\boldsymbol{x}_{rd^*}$ is the alternative which was used to determine the acceptable value of utility r (see (4)).

Figure 1(b) illustrates the situation in which both reference alternatives yield the same utility value r. This is not necessary and there may be situations when there is only one such alternative, yet two or more reference alternatives need to be selected. In such case alternatives utility value is greater, but differ from r as little as possible, can be selected to generate *r-acceptable sets.* We propose to select alternatives which utility is as close to r as possible so that no alternative that is significantly better for the buyer is removed. Only these alternatives which are marginally better can be removed.

Any choice of the number of reference alternatives influences information I_t the bidders receive at the beginning of round t. An increase of reference alternatives expands the set of r-acceptable alternatives for the next round by alternatives dominated from owner's point of view and it may encourage bidders to submit new offers which otherwise would be excluded.

In each round t information I_t about r-acceptable sets is presented to the bidders. Therefore the notation X_{rD}^{At} is used to describe set of r-acceptable alternatives which is a subset of X formulated for round t and defined by D reference alternatives (t = 0, 1, …, T; $D < L$). To simplify the notation, the r value, number of reference alternatives and/or the round number, are dropped when unnecessary.

Note that the assumed monotonicity of utility function allows us to use single reference alternative $\boldsymbol{x}_r$ for the construction of the r-acceptable set X_{D1}^{At} for t = 0, 1, …, T). If the utility is non-monotonic and reaches one or more optimum within the set rather than on its boundary, then the acceptable set needs to be defined by more points.

Parameter D, which defines the number of reference alternatives used to construct the r-acceptable set, is one of the *mechanism design parameters*. The buyer needs to determine its value and this requires taking into account the following two types of tradeoffs:

Tradeoff 1. The relationship between the number of alternatives which are r-feasible but not included in r-acceptable set X_D^{At} and the bidders' difficulty in selecting bids from this set. The greater the value of D, the fewer acceptable alternatives are not included but the number of sets in which the bidders need to consider increases making bidding more difficult.

Tradeoff 2. The relationship between the number of alternatives D used to specify X_D^{At} and the bidders' ability to discover the buyer's utility function. The greater the D value the easier it is to determine the analytical form of the utility.

The second type of tradeoff should be addressed because D often has to be greater than the minimum number of alternatives required to determine the owner's utility function. In the example shown in Figure 1(b) the selection of two reference alternatives would allow the bidders to determine the buyer's utility function. Reduction of the number of reference alternatives from two to one leads to the situation described in Figure 1(a), in which many acceptable alternatives are excluded.

3.4.2 Perturbation of *r*-acceptable set

Design parameter D is used to control the construction of such an r-acceptable set, that excluded r-feasible alternatives are not sufficiently relevant for the buyer. The buyer knows that some alternatives are excluded but they do not differ much from some of the alternatives which are then included.

An increase of the value of D expands the set but, as Tradeoff 2 above states, it does so at the cost of increasing the possibility of preference disclosure. In general, it is not possible to avoid indirect disclosure of the owner's preferences and keep all r-feasible alternatives in the r-acceptable set (i.e., these that yield equal or higher utility than the winning bid). If, however, the owner accepts that some r-feasible alternatives are excluded, then a disclosure can be avoided by making a small change in some of the reference alternatives. This situation is illustrated in Figure 2.

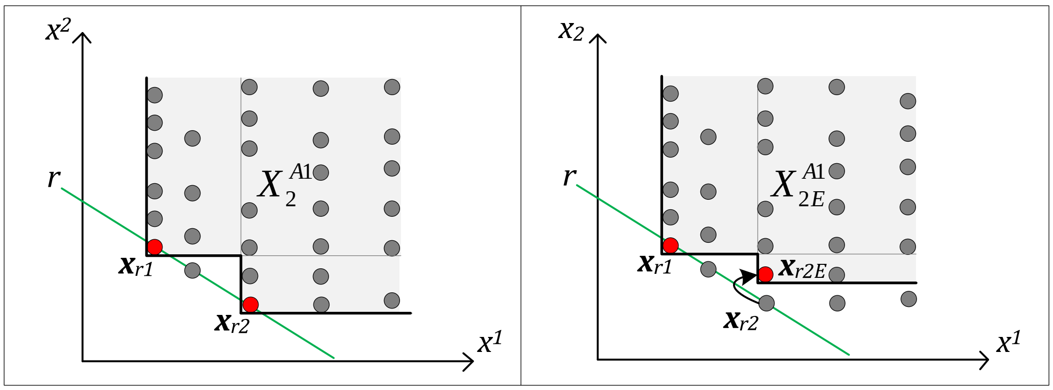

Figure 2. (a) Acceptable set with two-point reservation levels; and (b) the same set with a single-point perturbation.

To address the issue of the discovery of the buyer's utility function, a deviation from the reference alternatives may be introduced. An example of this intervention is presented as the transformation of set X_2^A shown in Figure 2(a) to set X_{2E}^A shown in Figure 2(b). The intervention is the replacement of the reference alternatives. In our example, alternative x_{r2} is replaced with $\boldsymbol{x}_{r2E}$. The result of this perturbation may be a loss of some of the r-feasible alternatives but the bidders' ability of discovering the buyer's utility is diminished. In effect we have the third type of tradeoff:

Tradeoff 3. The relationship between the deviation size is associated with the number of removed acceptable alternatives and the increased difficulty in discovering the buyer's utility function. The greater the deviation, the greater the difficulty but this comes at the expense of an increase in the number of acceptable alternatives removed.

We use parameter E to control the size of the deviation. E is the second mechanism design parameter and its value also needs to be decided externally to the procedure and it reflects the buyer's willingness to accept a winning bid that is not Pareto-optimal in X.

E may be applied in a number of ways. For example, from the initial list of D reference alternatives which have utilities equal to the winning bid utility, select about 50% of the D alternatives and increase the value of one attribute in each alternative. The selected attributes should be different and their selection should be such that the utility of thus constructed reference alternatives differs as little as possible from the winning bid utility.

The application of parameter E allows the replacing set of r-acceptable alternatives X_D^{At} with the subset X_{DE}^{At}, i.e.,

$$X_{DE}^{At} \subset X_D^{At}. \quad (9)$$

To indicate that X_{DE}^{At} is a subset of r-acceptable set obtained by the application of parameter E, we say that its elements are r_E-acceptable alternatives.

The two design parameters D and E operate in opposite ways. Parameter D is used to expand the r-acceptable set so that none or a few r-feasible alternatives are excluded. Parameter E contracts the r-acceptable set so that the bidders cannot determine the buyer's utility through fitting a curve to the reference points $\boldsymbol{x}_{rd}$ (d = 1, …, D).

4. Process

In this section multiattribute procedure for reverse auctions is presented. The construction of r-feasible and r-acceptable sets is at its core.

4.1 Preliminaries

During the auction, the construction of these sets relies on reference alternative $\boldsymbol{x}_r$, which utility is $r(u(\boldsymbol{x}_r) = r)$. Before the auction begins we need a way to construct the first r-acceptable X_{DE}^{A0} set and present it to the bidders. There may be different ways to construct this set. For example, the owner may use initial reservation levels, the minimum acceptable utility value, or propose the feasible set X.

Figure 3(a) illustrates the initial auction state in round t=1. The r-acceptable set is defined by two alternatives $\boldsymbol{x}_{r1}$ and $\boldsymbol{x}_{r2}$. In this round three bids were made. These are shown in Figure 3(b). Among the three bids $\boldsymbol{x}_{r2}^2$, yielded the highest utility value u_2. Therefore u_2 and $\boldsymbol{x}_{r2}^2$ are used to determine the r-acceptable set for round t+1. This new set is shown in Figure 3(b).

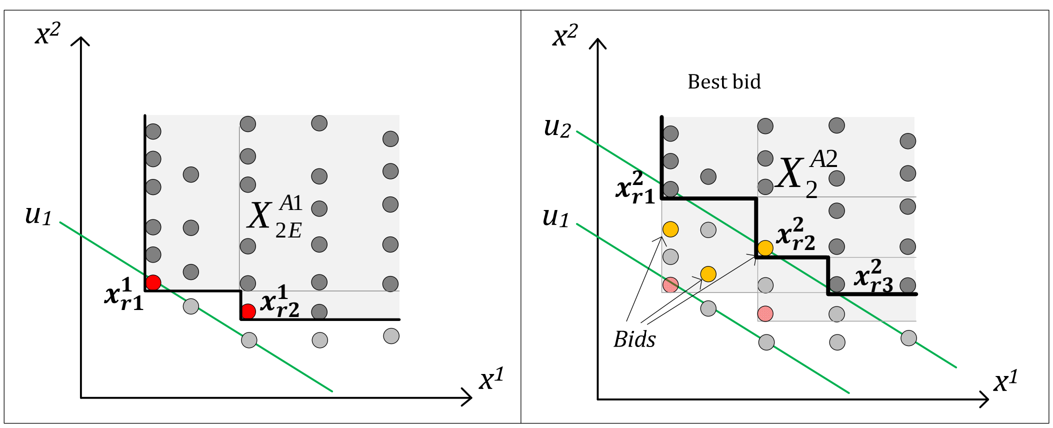

Figure 3. (a) Initial acceptable set; and (b) revised acceptable set after 3 bids.

The two design parameters D and E have a critical role in the procedure and its convergence. Their values may be constant or changed during the auction according to a predefined formula. In each case, the values which are used in round t need to be verified for their feasibility. The reason for the verification of D is that there may be no alternative which can be used as a reference alternative given by (6). That is, after d = 1,…, d_1 alternatives were selected, there may be no alternatives $\boldsymbol{x}_{rd}, (d = d_1 + 1, \dots, D)$, such that the following two conditions are met:

(i) $U(\boldsymbol{x}_{rd}) \geq r$; and
(ii) (ii) $\boldsymbol{x}_{rd} \notin \cup_{d=1}^{d_1} X_{rd}^A$ (10)

Condition (i) above is a part of the construction of the r-feasible set. Condition (ii) needs to be met because sets are convex cones X_{rd}^A, (see (4)). Therefore, if a reference alternative $\boldsymbol{x}_{ra}$ is selected, which is already an element of r-feasible set X_{rb}^{A}, ($1 \le b < a \le D$), then $X_{ra}^{A} \subset X_{rb}^{A}$. This means that set X_{ra}^{A} is redundant and may be removed.

The value of the deviation control parameter E may also need to be verified when it is greater than the smallest allowable increment. If the value is greater, then its application may result in the modification of set X_{rd}^{A} to set X_{rdE}^{A} which removes all r-acceptable alternatives so that X_{rdE}^{A} is empty.

Note that the value of parameter E may depend on the character of an attribute. For example, the value will be different for attribute describing, price, warranty and delivery time. In such situations we denote E^n as the deviation control parameter of attribute n (n= 1, …, N).

Given the above caveats, we can simplify the description of the procedure and assume that the values of the two parameters are held constant and that they need not be changed during the auction. To stress the procedure reliance of the two parameters and the assumption that their values are held constant, we denote the procedure $\mathbb{P}_{DE}$.

4.2 Procedure

Procedure $\mathbb{P}_{DE}$ comprises the following ten steps:

1. Determine value D. If, for each attribute, deviation values E^n are different from the smallest meaningful increment of attribute ε^n, then determine values E^n (n = 1, …, N).
2. Determine the number N^u of points sufficient to define utility function u.
3. Set t = 0, construct initial set X_{DE}^{A0} , and request bids.
4. Set $t = t + 1$. If there are no bids, then terminate, otherwise move to Step 5.
5. Select the best bid $\boldsymbol{x}^{t*}$ made in round t and set $u(\boldsymbol{x}^{t*}) = r^t$).
6. Select D-1 reference alternatives $\boldsymbol{x}_d$, such that, $u(\boldsymbol{x}_d) = r^t$, d= 1, …, D-1. If the number d_r of these alternatives is smaller than D-1, then select alternatives with utility as close to r as possible, but not smaller than r^t.
7. Use formula (7) to construct set X_{D}^{At}
8. If $D\text{-}1\text{-}d_r > 0$ (i.e., D-1-d_r alternatives which yield utility greater than r were added in Step 6), then check if $d_r + 1 \ge N^u$, i.e., if it is possible that d_r +1 alternatives can uniquely define utility function u. If not, then set $X_{DE}^{At} = X_{D}^{At}$ and go to Step 10.
9. Construct set X_{DE}^{At}. Set $D^e = d_r - N^u$. Select reference alternative $\boldsymbol{x}_d$ such that $u(\boldsymbol{x}_d) > r^t$, d = 1, …, D^e. For each alternative $\boldsymbol{x}_d$ select one attribute; begin with low preference attributes and continue with higher preference attributes. If the number of attributes $N < D^e$, repeat the cycle of selecting attributes. For each alternative $\boldsymbol{x}_d$ add value E^j to the value of the selected attribute j, d = 1, …, D^e.
10. Present set X_{DE}^{At} to the bidders and request bids. Go to Step 4.

4.3 Efficiency

In Section 1 we defined efficient alternatives. Correspondingly, we define auction mechanism efficiency in terms of the existence of efficient alternatives. Hence, a mechanism is efficient if its rules do not remove any efficient alternative. For procedure $\mathbb{P}_{DE}$ this means every efficient alternative which is an element in X is also an element in sets X_{DE}^{At}, t =0, …, T.

One of the roles of parameters, D and E, is to control the degree of procedure $\mathbb{P}_{DE}$ inefficiency, which is the difference between the winning bid and the utility value of an efficient alternative.

From (7) it follows that $\supset\ X_{D}^{At}$, t =0, …, T. Using (9) we obtain decreasing family X_{DE}^{A0}, t=1,…, T:

$$X \supset\ X_{DE}^{A0}\ \supset \cdots\ \supset X_{DE}^{AT-1}\ \supset X_{DE}^{AT}\ . \qquad (11)$$

Proposition: Formula (11) states that procedure $\mathbb{P}_{DE}$ reduces the number of alternatives which the bidders need to consider. Using (3), (4) and (6) we can formulate a sufficient condition for X_{DE}^{AT}, (given by (11)), to include efficient solution, which is:

$$u_0 < u_1 < \cdots < u_{T1-1} < u_T \tag{12}$$

where $u_t = u(\boldsymbol{x}^{t*})$, $\boldsymbol{x}^{t*}$ is the best bid made in round t.

The proof is straightforward – the two-dimensional version is demonstrated in the Figure 2.

Theorem 1. If parameter D, defined above, satisfies inequality

$$\forall\, \boldsymbol{x} \in X_{DE}^{At}: u(\boldsymbol{x}) \geq u_t(\boldsymbol{x}), \tag{13}$$

and set

$$X_{DE}^{At} = X_{DE}^{At-1} \backslash \{\boldsymbol{x}: u(x) < u(\boldsymbol{x}^{t*})\} \neq \emptyset, \tag{14}$$

then procedure $\mathbb{P}_{DE}$ does not remove any efficient alternative.

Proof: The proof results from a contradiction: An efficient alternative is one which utility is not smaller than $u(\boldsymbol{x}^{t*})$. If such an alternative is removed so that it is not an element of X_{DE}^{At}, then formula (13) for the construction of X_{DE}^{At} is not followed. If (14) is obeyed, then only alternatives which are worse for the buyer than the best bid $\boldsymbol{x}^{t*}$ are excluded.

Formula (12) assures that during the construction of r-acceptable set X_{DE}^{At} no acceptable alternative (i.e., one which meets the utility condition (4)) is removed. Condition (13) assures that only alternatives which utility is lower than the best bid utility may be removed when r-acceptable sets are constructed.

Definition: Bid $\boldsymbol{x}^*$ made in round T is winning if:

1. $X_{DE}^{AT+1} = \emptyset$; and
2. $u(\boldsymbol{x}^*) = u(\boldsymbol{x}_{j*}) \geq u(\boldsymbol{x}_j), j = 1, \ldots, J_T$.

Assuming that there is at least one bid in the auction governed by the process described by Theorem 1, then this process is convergent.

5. Discussion

The design parameters D and E contribute to the process complexity and efficiency as well as the possibility of the sellers discovering the buyer's utility. If D is large, the process is complex because the buyer conveys information about many r-acceptable sets and the process efficiency decreases. If E is large, then the efficiency increases and fewer rounds are required but the possibility of removing efficient alternatives from the r-acceptable sets increases.

In general, there is no solution that would allow for a simple and efficient process in which no efficient alternative is removed and the ability of the sellers cannot determine the buyer's utility. In this section we propose two procedural tactics which can be implemented for monotonic utility functions.

5.1 Distance-minimizing strategy

Utility theory posits that the buyer is not interested in the particularities of an alternative but in the alternative's utility value. The buyer may, however, be interested in the efficiency of the process. The efficiency may be increased if the procedure directs the bidders towards the shortest path from any given point to the ideal alternative $\boldsymbol{x}_M$.

Figure 4 illustrates the case when the best bid $\boldsymbol{x}^{t*}$ was made in round t. According to (12), the utility reservation level in round t+1 becomes $u_{t+1} = u(\boldsymbol{x}^{t*})$.

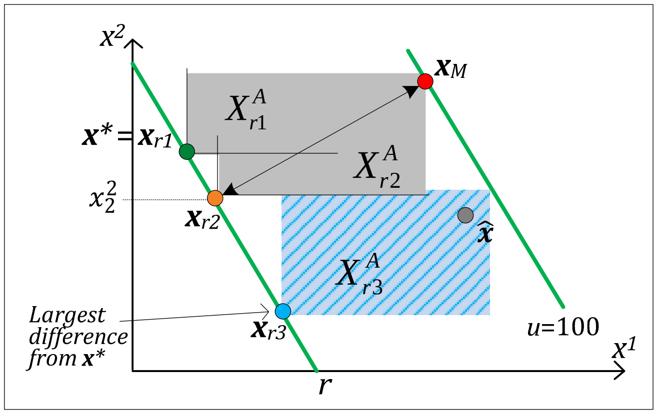

Figure 4. Best bid $\boldsymbol{x}^*$, ideal alternative,
and reservation levels for strongly non-uniform preferences

The distance-minimizing strategy means that an alternative where utility is u_{t+1} and which is the closest to $\boldsymbol{x}_M$ becomes the reservation point $\boldsymbol{x}_{r1}$ used to construct r-acceptable set X_{r1}^A. The buyer may wish to reduce the number of acceptable alternatives that are not included in this set and add the winning bid thus constructing set X_{r1}^A as illustrated in Figure 4.

5.2 Structurally different alternatives

There are situations when a buyer's preferences are strongly non-uniform. Some attributes are much more important to the buyer than others. In a two-dimensional space, this situation is illustrated in Figure 4; the angles of utility isoquants are significantly different so that x^1 is approximately twice as important as x^2.

The best bid ($\boldsymbol{x}^*$) may be relatively close to the ideal point so that there are many alternatives not included in the set $X_r^A = X_{r1}^A \cup X_{r2}^A$. This is shown in Figure 4. Limiting the bids to the union of sets X_{r1}^A and X_{r2}^A may result in removing a large part of the r-feasible alternatives (i.e., all alternatives yielding utility not lower than r). This may lead to a loss of efficient alternatives when, for example, there is a seller who bid $\widehat{\boldsymbol{x}}$, and no other seller made a bid yielding more than $u(\widehat{\boldsymbol{x}})$.

In order to avoid this situation, we propose to use alternatives where utility is equal or close to r but where they are significantly different from the best bid $\boldsymbol{x}^*$. To determine the structural difference between elements of set X, we map X onto N-dimensional space of natural numbers $\Im$, i.e., φ: X $\rightarrow \Im_X$, $\Im_X \subset \Im$, such that:

$$\varphi(\boldsymbol{x}_1, \boldsymbol{x}_l) = \begin{bmatrix} x_1^1 \\ \dots \\ x_1^N \end{bmatrix} - \begin{bmatrix} x_l^1 + i_l^1 \varepsilon^1 \\ \dots \\ x_l^N + i_l^N \varepsilon^{1N} \end{bmatrix} = \begin{bmatrix} i_l^1 \\ \dots \\ i_l^N \end{bmatrix} = \boldsymbol{i}_l \in \Im_X, \tag{15}$$

where: $\boldsymbol{x}_1$ is the alternative which attributes take the smallest values as defined in (2) or, for the nominal attributes, it is the least preferred value;

ε^n is the smallest meaningful increment of attribute x^n value(n = 1, ..., N) or , for the nominal attributes, the increment is equal to 1; and

i_l^n is the number of increments that the attribute value x_l^n differs relative to its smallest value x_1^n or, for the nominal attributes, it is the number of attribute values which are preferred, less than value x_l^n.

Definition: The difference δ between two alternatives $\boldsymbol{x}_k$ and $\boldsymbol{x}_l$ is the sum of absolute differences between the attribute values of the corresponding points $\boldsymbol{i}_k$ and $\boldsymbol{i}_l$. That is:

$$\delta_{kl} = \sum_{n=1}^{N} |i_k^n - i_l^n|. \tag{16}$$

The tactic proposed to avoid a situation in which many *r*-feasible alternatives are not included in *r*-acceptable sets (see Figure 4) is based on the selection of δ* such that:

$$\delta^* = \max_{\{l:\, u(\boldsymbol{x}_l) \geq u(\boldsymbol{x}*)\}} \delta_{*,l} \tag{17}$$

In the example illustrated in Figure 4, the largest difference computed with (17) corresponds to point $\boldsymbol{x}_{r3}$.

5.3 Auction rounds and closure

We consider auctions which have *T* rounds. An auction may end in a *failure* if there was no bid in the first round (t=1). If *T* is known from the outset, then an auction may end *earlier*, when no bid was made in round t, $t < T$.

The buyer may want to control the process and decide about the minimum utility value increment. The increment $\Delta u_{t+1} = u_{t+1} - u_t$, defined by (12), is sufficiently small so that no acceptable alternative that could be a winning bid is removed. If the buyer increases the minimum increment, then the process' efficiency may increase and fewer rounds are required. The downside of such increment change is that the winning bid may be inefficient. This means that minimum utility increment, denoted as Δ_u, constitutes a process-defining parameter.

Figure 5 illustrates the move from round t to round t+1. In Figure 5(a) the new minimum increment (u_{t+1}–u_t) does not remove any efficient alternative. Bid $\boldsymbol{x}_{r2}$ made in round t, is an element of the *r*-acceptable set and only alternatives which this bid dominates are removed. This process follows formulae (13) and (14).

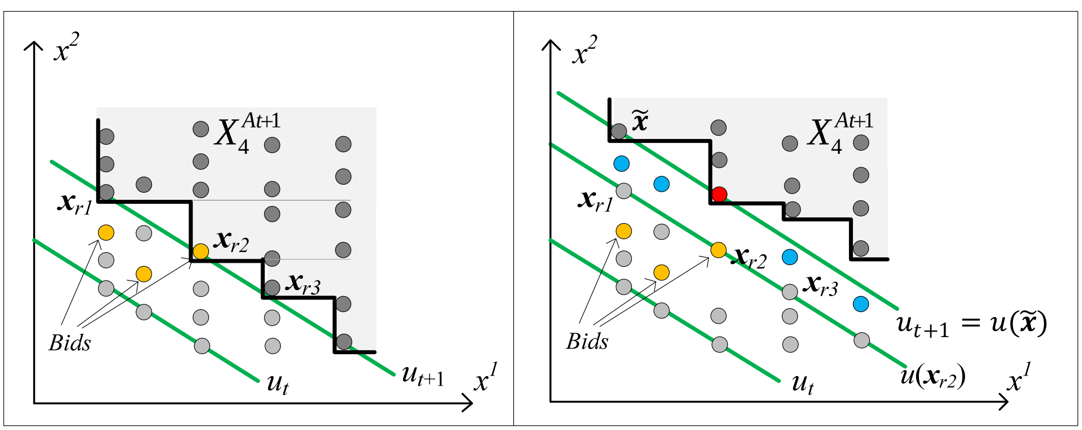

Figure 5. Two revised acceptable sets: (a) all alternatives dominating $\boldsymbol{x}_{r2}$ are included; and (b) some alternatives (in blue) are excluded.

Consider the situation when the minimum utility increment is greater than Δu_{t+1} defined by (12). The reason for setting a higher increment may be the buyer's need to get the best bid by the self- or externally-imposed deadline or within a given number of rounds. In the case of a fixed length auction the minimum increment may be defined by:

$$\Delta_u = (u(\boldsymbol{x}^M) - u_0) / \mathrm{T} \tag{18}$$

where: u_0 – the minimum acceptable utility value (used to construct the initial set X_{DE}^{A0} described in Step 3 of procedure $\mathbb{P}_{DE}$).

Actual bids are not taken into account in (17), making it not useful when, for example, the best bid utility in one round significantly exceeds the increment required in the next round.

Increment can also be a function so that initially it takes a large value which with each subsequent round decreases (e.g. exponential Brigui-Chtioui and Pinson 2010). Because the bidders are likely to make bids that exceed the utility reservation value, an adaptive rule which is a generalization of (18) can be used:

$$\Delta_{ut} = \max\{\, \Delta_u; (u(\boldsymbol{x}^M) - u(\boldsymbol{x}^{t*}))/(T - t + 1), \tag{19}$$

where: $\boldsymbol{x}^{t*}$ – the best bid obtained in round t, $t = 1, \dots, T - 1$.

Taking into account utility of bids' allows using the minimum utility increment only when it is necessary. Formula (18) does not require changing procedure $\mathbb{P}_{DE}$, (see Section 4.2) as long as the biding process progresses according to the buyer's expectations, that is the best bid utility exceeds minimum utility increment, that is:

$$u(\boldsymbol{x}^{t*}) \geq u_t + \Delta_{ut}, t = 1, \dots, T - 1. \tag{20}$$

If, however, the best bid utility is lower, that is inequality (20) does not held, then the minimum increment Δ_{ut} is introduced. Because the utility of bid $\boldsymbol{x}^{t*}$ is lower than required $u_t + \Delta_{ut}$another reference point has to be introduced. Let $\widetilde{\boldsymbol{x}}^t \in X_{DE}^{At}$ be the reference point replacing $\boldsymbol{x}^{t*}$. Following this replacement, Step 5 in $\mathbb{P}_{DE}$, is modified as follows:

> Step 5 (revised). Select the best bid $\boldsymbol{x}^{t*}$ made in this round and apply (18). If $\Delta_u < \Delta_{ut}$, then go to Step 6, otherwise replace $\boldsymbol{x}^{t*}$ with $\widetilde{\boldsymbol{x}}^t$ and set: $r = \widetilde{\boldsymbol{x}}^t$.

The situation in which (20) does not held is shown in Figure 5(b). In round t, the best bid is $\boldsymbol{x}_{r2}$ but $u(\boldsymbol{x}_{r2}) < u_t + \Delta_{ut} = u_{t+1}$. Alternative $\widetilde{\boldsymbol{x}}$ with utility equal to u_{t+1}. Additional reference points are selected and set X_4^{At+1} is constructed. In the situation when there are no more bids, $\boldsymbol{x}_{r2}$ becomes the winning bid. This bid, however, may be inefficient because we cannot exclude the possibility the bidders could make a bid that yields utility higher for the owner than $\boldsymbol{x}_{r2}$ and lower than $\widetilde{\boldsymbol{x}}$. The bidders could choose one of the alternatives shown in blue that dominate $\boldsymbol{x}_{r2}$, but they were unable to do so by the set of constraints they had to obey.

6. Example

We illustrate the proposed reverse auction procedure with the example used by (Bellosta, Kornman et al. 2008, p. 403).

Consider a buyer who wants to purchase a car. There are three attributes (N=3) that she car is interested in: trademark, warranty, and price. Trademark is a nominal attribute. Warranty and price are numerical attributes; the minimum increment for guarantee is one month and for price it is $1. In Table 1 the attributes and their values (ranges) are shown.

Table 1. Value functions

Attribute	Trademark						Warranty (month)		Price ($)	
Weight of attribute (v_j)	$v_1 = 0.3$						$v_2 = 0.2$		$v_3 = 0.5$	
Attribute values (x_j^i)	Star	Citron	Pejo	Betha	Reno	Lux	0	60	10,000	50,000
Weight of attribute value (w_j^i)	0	20	40	60	80	100	0	100	100	0

The attribute and the attribute value partial utilities are shown in Table 1. For the two numerical attributes partial utilities are defined by a linear function of the form:

For warranty values: $w_l^2 = (50{,}000 - x_l^2)100/(50{,}000 - 10{,}000)$; and

For price values: $w_l^3 = x_l^2)100/60$.

$w_l^j = 100x_l^j/(A - B)$, where A is the maximum and B the minimum value of attribute j (j=2,3).

Consider two alternatives: $\boldsymbol{x}_1$ = [Reno; 36; 40,000] and $\boldsymbol{x}_2$ = [Pejo, 24, 30,630], whose corresponding attribute values are (80, 60, 25) and (40, 40, 48), respectively (See Table 2).

Table 2. Two alternative examples

	Alternative 1 ($\boldsymbol{x}_1$)			Alternative 2 ($\boldsymbol{x}_2$)		
	x_1^j	w_1^j	u^j	x_2^j	w_2^j	u^j
Trademark	Reno	80	24	Pejo	40	12
Warranty	36	60	12	24	40	8
Price	40000	25	12.5	30630	48	14
Total	$u(\boldsymbol{x}_1) = 48.5$			$u(\boldsymbol{x}_2) = 44$		

Because u_1= 48.5 and = 44, $u_1 > u_2$, x_1 is preferred to x_2.

The buyer announces a reverse auction in which three sellers participate. The buyer informs the bidders that:

1. Every trademark except for start is acceptable;
2. The warranty cannot be lower than 12 months;
3. The maximum price is $50,000

From Table 1, we obtain that:

$u(r(0))$ = 0 and $x_r(0)$ = (Star, 0, 50000)

$u(x_M)$ = 100 and $a = x_M$ = (Lux, 60, 10000)

Assume that the buyer announces 10 rounds for this auction T=10.

$\Delta_u(0) = (100\text{-}0)/10 = 10$

Construction of r-acceptable sets:

The criteria for the three constraints are:

C1: (40, 0, 0), and $u(c_1) = 12 \geq 10 + 0$

C2: (0, 50, 0), and $u(c_2) = 10 \geq 10 + 0$

C3: (0, 0, 20), and $u(c_3) = 10 \geq 10 + 0$

Round 1

The constraints announced to the bidders for the 1st round are:

C1: $x^1 \in$ {Pejo; Betha; Reno; Lux}; $x_2 \geq 0$ months, $x_3 \leq$ \$50,000; or

C2: $x^1 \in$ {Star; Citron; Pejo; Betha; Reno; Lux}; $x^2 \geq 30$ months, $x^3 \leq$ \$50,000; or

C3: $x^1 \in$ {Star; Citron; Pejo; Betha; Reno; Lux}; $x^2 \geq 0$ months; $x^3 \geq$ \$42,000

Receive bids in Round 1.

S1: (20, 100, 25) as (Citron, 60, 40000)

S2: (40, 100, 25) as (Pejo, 60, 40000)

S3: (60, 100, 10) as (Betha, 60, 46000)

Select the best bid in Round 1.

Because $u_{s2} > u_{s3} > u_{s1}$, x_{s2} is selected as the best bid in round 1,

i.e. $x_{k^*}(1)$ = (Pejo, 60, 40000) and $u_{k^*}(1) = 44.5$.

Set step for Round 2.

$\Delta_u(1) = (100\text{-}44.5)/(10\text{-}1) = 6.2$

Set new reservation points *r*(*1*).

Find the point $r(1) = (56, 71, 27)$,

which satisfies $u(r(1)) = u_{k^*}(1) = 44.5$ and $|| r(1), u_M|| = 39$ (i.e. the shortest distance)

while the closet feasible alternative is $x_r(1)$ = (Citron, 42, 39211)

Formulate constraints.

The criteria for the three constraints are:

C1: (60, 71, 37), and $u(c_1) = 50.7 > 44.5 + 6.2$

C2: (40, 100, 38), and $u(c_2) = 51 > 44.5 + 6.2$

C3: (60, 96, 27), and $u(c_3) = 50.7 > 44.5 + 6.2$

The constraints announced to the bidders for the 2nd round are:

C1: $x_1 \geq$ Betha, $x_2 \geq 43$ months, $x_3 \leq \$35200$; or

C2: $x_1 \geq$ Pejo, $x_2 \geq 60$ months, $x_3 \leq \$34800$; or

C3: $x_1 \geq$ Betha, $x_2 \geq 58$ months, $x_3 \geq \$39200$.

The auction repeats until there is only one bidder left.

7. References

Aaron, M. C. (1995). "The Value of Decision-analysis in Mediation Practice." Negotiation Journal **11**(2): 123 - 133.

Anderson, J. and M. Frohlich (2001). "FreeMarkets and Online Auctions." Business Strategy Review **12**(2): 59-68.

Beil, D. and L. Wein (2003). "An Inverse-optimization-based Auction Mechanism to Support a Multiattribute RFQ Process." Management Science **49**(11): 1529-1545.

Bellosta, M., S. Kornman, et al. (2008). "A Unified Framework for Multiple Criteria Auction Mechanisms." Web Intelligence and Agent Systems **6**(4): 401-419.

Bellosta, M. J., I. Brigui, et al. (2004). A Multicriteria Model for Electronic Auions. ACM Symposium on Applied Computing (SAC'04).

Bichler, M. (2001). The Future of E-markets: Multidimensional Market Mechanisms. Cambridge Cambridge University Press.

Bichler, M. and J. Kalagnanam (2005). "Configurable Offers and Winner Determination in Multi-attribute Auctions." European Journal of Operational Research **160**(2): 380-394.

Block, C., J. Collins, et al. (2010). Exploring Retail Energy Markets through Competitive Simulation. ACM EC 2010 Workshop on Trading Agent Design and Analysis, Harvard University.

Brigui-Chtioui, I. and S. Pinson (2010). A Variable Bid Increment Algorithm for Reverse English Auction. Progress in Artitficial Economics, Springer.
Burmeister, B., T. Ihde, et al. (2002). A Practical Approach to Multi-attribute Auctions. 13th Workshop on Database and Expert Systems Applications (DEXA), IEEE.
Che, Y.-K. (1993). "Design Competition through Multidimensional Auctions." RAND Journal of Economics **24**(4): 668-680.
Dieckmann, A., K. Dippold, et al. (2009). "Compensatory versus Noncompensatory Models for Predicting Consumer Preferences." Judgment and Decision Making, vol. 4, no. 3, April 2009, pp. 200-213 **4**(3): 200-213.
Elmaghraby, W. (2004). Handbook of Quantitative Supply Chain Analysis: Modeling in the E-Business Era. Handbook of Quantitative Supply Chain Analysis: Modeling in the E-Business Era. D. Simchi-Levi, D. Wu and M. Shen. Norwell, Kulwer.
Engel, Y. and M. P. Wellman (2010). "Multiattribute Auctions based on Generalized Additive Independence." Journal of Artificial Intelligence Research **37**(1): che479-526.
Engelbrecht-Wiggans, R., E. Haruvy, et al. (2007). "A Comparison of Buyer-determined and Price-based Multi-attribute Mechanisms." Marketing Science **26**(5): 629–641.
Fishburn, P. C. (1976). "Noncompensatory Preferences." Synthese **33**: 393-403.
Jap, S. (2003). "An Exploratory Study of the Introduction of Online Reverse Auctions." Journal of Marketing **67**(3): 96-107.
Katok, E. and A. Wambach (2011). "Collusion in Dynamic Buyer-Determined Reverse Auctions." Management Science: forthcoming.
Kittsteiner, T. and A. Ockenfels (2006). "Market Design: A Selective Review." Zeitschrift für Betriebswirtschaft **5**: 121-143.
Milgrom, P. and R. Weber (1982). "A Theory of Auctions and Competitive Bidding." Econometrica **50**(5): 1089-1122.
Parkes, D. C. and J. Kalagnanam (2005). "Models for Iterative Multiattribute Procurement Auctions." Management Science **51**(3): 435–451.
Peleg, B. (2003) "The Value of Procurement via Online Bidding." Whitepaper, 3.
Pontrandolfo, P., S. Wu, et al. (2010). Auctions and Negotiations in Transportation Service Procurement. Group Decision and Negotiations.
Schoenherr, T. and V. A. Mabert (2007). "Online Reverse Auctions: Common Myths versus Evolving Reality." Business horizons **50**(5): 373-384.
Strecker, S. and S. Seifert (2004). Electronic Sourcing with Multi-attribute Auctions. HICSS 37, Hawaii.
Teich, J. E., H. Wallenius, et al. (1999). "Multiple Issue Auction and Market Algorithms for the World Wide Web." Decision Support Systems **26**: 49–66.
Teich, J. E., H. Wallenius, et al. (2004). "Emerging Multiple Issue E-auctions." European Journal of Operational Research **159**(1): 1-16.
Wagner, S. M. and A. P. Schwab (2004). "Setting the Stage for Successful Electronic Reverse Auctions." Journal of Purchasing and Supply Management **10**(1): 11-26.
Walley, M. and D. Fortin (2005). "Behavioral Outcomes from Online Auctions: Reserve Price, Reserve Disclosure, and Initial Bidding Influences in the Decision Process." Journal of Business Research **58**(10): 1409-1418.
Yu, B., G. E. Kersten, et al. (2008). Multi-attribute Auctions and Negotiations in Service Procurement: An Experimental Comparison. Group Decision and Negotiation Conference.