Selling Through Priceline? On the Impact of Name-Your-Own-Price in Competitive Market

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Priceline.com patents the innovative marketing strategy, Name-Your-Own-Price (NYOP), that sells opaque products through customer-driven pricing. In this paper, we study how competitive sellers with substitutable, non-replenishable goods may sell their products (1) as *regular* goods, through a direct channel at posted prices, and possibly at the same time (2) as *opaque* goods, through a third-party channel, which allows for the NYOP approach. We establish a stylized model that incorporates three sets of stakeholders: two competing sellers, an intermediary NYOP firm, and a sequence of customers. We first characterize customers' optimal purchasing/bidding decisions under various channels structures, and then analyze sellers' dynamic pricing equilibrium accordingly. The results highlight the impact of inventory and time on equilibrium prices, expected profit and channel strategy. We find that the implications can be very different for competing sellers under dual- versus single-channel structures. In particular, more inventory may reduce one's expected profit, which never happens when a seller uses direct channel only. Interestingly, although competing sellers do not always benefit from the existence of NYOP channel, it is possible that one or both of the sellers will still adopt it in equilibrium. We characterize areas for each type of channel structure to be in equilibrium. Finally, we conduct an experimental survey to validate some market-end assumptions, and discuss potential future extensions. To our best knowledge, this is one of the first studies that examine the impact of NYOP-opaque selling from an operations perspective.

Key words: Competition; Distribution channels; e-Commerce; Game Theory; Opaque Product; Pricing; Probabilistic Goods; Name-Your-Own-Price;

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1. Introduction

Decades since the emergence of e-commerce, online B2C shopping has entered a renaissance age in which sellers actively seek innovative techniques that increase flexibility of their offerings and operations. These innovations have taken place in: (1) product/service offerings (e.g., opaque products offered by Hotwire.com and Priceline.com as well as many tour operators), (2) pricing (e.g., customer-driven pricing at eBay.com and Priceline.com), and (3) transaction process (e.g., total quantity threshold at Groupon.com). In making products, prices and business transactions flexible, these innovations ease capacity tension, refine market segmentation, and hedge for uncertainties in both supply and demand, which ultimately enhances the efficiency of the system.

One of the critical components in these innovations is the platform (e.g., Hotwire, Priceline, eBay, Groupon) which acts as the liaison between the product/service providers (e.g., hotels, manufactures) and the customers. Successful platforms need to complement the traditional operations of the product/service providers, and be rewarding to their hosting firms at the same time. For example, Priceline.com, which patented the “name-your-own-price” (NYOP) mechanism, has thrived ever since the dot-com bubble. With the 2008 economic crisis raising the wave of price-conscious customers, Priceline.com has overtaken its competitor Expedia.com in market capitalization (Tnooz.com, Nov 17, 2009), and then in gross bookings (Skift.com, Nov 8, 2013). The 2011 Priceline.com Annual Report also confirms that it is the booming NYOP sales that contribute to the company’s annual increase in both domestic and global businesses.

The mechanism that innovates along all three aforementioned dimensions (products, pricing and processes), the NYOP business model has not yet been fully studied in operations management literature. This paper aims at deepening the understanding of NYOP from an operations perspective, and offering insights into some fundamental questions.

To begin with, let us consider how NYOP works. Suppose a customer is looking for a round-trip air ticket from Los Angeles (LAX) to Montréal (YUL), departing on Feb 22 and returning on Feb 28. In acquiring such a trip via NYOP at Priceline, the customer needs to provide the departure and returning dates with respective airport codes, as well as the price that he/she is willing to pay. Priceline processes all the information and then notifies the customer whether his/her price is accepted. There are three distinctions/aspects of this process, namely:

- **Products**: there are more than one flights between LAX and YUL on the designated dates, and Priceline can assign the customer to any of them at its own discretion. That is to say, the customer will not learn the specifics of the trip (e.g., airlines, departure/arrival time, number of connections) until the transaction is finalized. In literature, such products whose characteristics are not fully revealed at the point of payment have been referred to as *opaque* goods (Anderson 2008, Fay 2008), as opposed to *regular* goods features of which are known to the buyers at the time of purchase.

- **Pricing:** different from the traditional *seller-driven* pricing, where a seller posts the price of the goods (also called tag price, or listed price) and a buyer simply makes a take-it-or-leave-it decision, in NYOP it is the buyer who initiates a price and the seller determines whether to release a unit of the goods at that price. Therefore, NYOP belongs to the family of *buyer-driven* pricing. Both types of pricing mechanisms can be applied to either opaque or regular products. Table 1 summarizes how these have been implemented in industry.

	Regular Goods	**Opaque Goods**
Seller-Driven	Expedia, Orbitz	Hotwire, BookIt
Buyer-Driven	Auction: eBay, Google NYOP: Unrevealed German NYOP firm (Hann and Terwiesch 2003, Terwiesch et al. 2005)	NYOP: Priceline Urbanoffer

Table 1 Pricing Schemes for Regular and Opaque Goods

- **Process**: using NYOP means commitment to buy. That is, once a customer's bid is accepted, he/she will be charged immediately, and the trip can no longer be altered (purchases through NYOP are non-refundable and non-changeable). In case of a rejection, the transaction is cancelled. Technically, the customer cannot make the same bid within the next 24 hours [1].

Given these distinctions, it is unclear how NYOP may affect product/service providers' business. On one hand, it may help to increase market penetration (as information-less opaque product can be sold at lower price), while on the other hand, it may cannibalize one's existing channel (customers may also find that buying at the posted price could be quite costly since there is a chance to acquire the *same* item at lower price through NYOP). Sellers, therefore, have to balance the NYOP-opaque channel and posted-price-direct channel in a competitive market. These features motivate our interest in studying strategic interactions among stakeholders in a supply chain setting. In particular, we investigate the impact of NYOP channel on market segmentation, pricing and expected profit, as well as the main drivers for the adoption of the NYOP channel.

Our results provide interesting implications to each stakeholder. On the market end, customers' channel and bidding strategies depend on the channel structure and valuations of the products. For the competing sellers, equilibrium prices and profits can differ significantly under various channel structures. In particular, sellers' expected profits are generally lower when they adopt NYOP channel, and the expected profit can even be decreasing in the inventory levels, which never happens if at least one seller uses direct channel only. Interestingly, although the existence of NYOP channel does not always benefit the sellers, in equilibrium both sellers can adopt NYOP channel at the same time. In general, equilibrium channel structure can take many forms and we identify areas for each of them in the numerical study. Finally, we show that for the intermediary NYOP firm, horizontally differentiated products are usually more beneficial than vertically differentiated ones. Thus, proper opaque product design, or supplier base construction, is paramount to the intermediary firm.

[1] Priceline may encourage customers to revise certain conditions *and* increase their prices in qualifying for a second bid. Yet the main tone is that one cannot repeatedly bid for the *same* item.

The model presented in this paper adds a new block to the current study of NYOP: (1) We study dual channel–dynamic pricing strategy in a competitive market. Specifically, we allow the competing firms to adopt one or two channels throughout the time. The value of the NYOP channel is then analyzed in a more concrete manner; to the best of our knowledge, such treatment of the problem received only limited attention in relevant literature. (2) Different from many existing NYOP studies, which are mainly customer-behaviour oriented, our focus is more on the supply side. This emphasis has practical implications for industries where NYOP applies. We believe that these distinctions make the problem/model a relevant and interesting research topic.

The paper is organized as follows. We review the literature in §2 and introduce the model in §3. §4 studies customers' purchasing and bidding decisions under given prices and channel structures. §5 analyzes the optimal or equilibrium pricing of the sellers. Extensive numerical study is conducted in §6, revealing important insights into the impact of inventory and time on pricing, profit, and channel decisions. §7 validates some model assumptions via an experimental approach, and discusses some future extensions.

2. Literature Review

The business model of NYOP and opaque goods has drawn increasing attention in recent literature. In the early years, many papers on NYOP consider regular products only.[2] These papers are usually consumer-oriented, emphasizing consumer bidding behavior and/or restrictions that sellers may put on bidding (e.g., Hann and Terwiesch 2003, Fay 2004, Spann et al. 2004, Terwiesch et al. 2005, Hinz and Spann 2008, Fay and Laran 2009). However, most of the work assumes that NYOP is the only sales channel. Not until recently did researchers begin incorporating direct channels in the model. For example, Cai et al. (2009) study the value of double-bid option as well as direct channel with an NYOP seller. Wang et al. (2009) allow a firm to adopt NYOP only in last minute sales, and find out that the value for an NYOP channel depends on the uncertainty of high-fare demand, rather than on the amount of excess capacity.

While most of the NYOP works consider single seller only, problems involving opaque goods usually demand a very different model structure (Granados et al. 2010a), as it usually takes two or more sellers/products to provide an opaque offering. For this reason, competition may arise (for monopolistic opaque-product case, see Gallego and Phillips 2004, Jiang 2007, Fay and Xie 2008). In addition, the opaque product can be priced in a seller-driven (posted price) or buyer-driven (e.g., auction, NYOP) manner. Fay (2008) models selling opaque products at posted prices only. Conditions under which the opaque good may bring down the price in the traditional channel

[2] Some of the models do not particularly differentiate regular versus opaque products—we put them under the column "Regular Goods," while the second column contains papers targeting only opaque products.

and harm the revenues are identified. Jerath et al. (2010) consider a problem where firms can sell through a posted-price opaque channel in the last minute, and analyze respective benefit. Granados et al. (2010b) examine the market response to opaque product and identify conditions under which it may cannibalize existing market or generate new demand. Granados et al. (2011) further provide evidence upon the price-information trade-off within the opaque channel.

If we consider NYOP for pricing, the notion of "NYOP firm" needs some further clarification. In the presence of opaque goods, the NYOP system usually consists of an NYOP intermediary firm and a number of sellers, in which the former acts as an intermediary that delegates the NYOP service for the latter. This is different from many aforementioned papers, which deal with only one "NYOP seller"—a centralized seller that also provides NYOP service. With opaque products, the sellers and NYOP intermediary are usually separated. The sellers determine their reservation prices, and the NYOP intermediary selects the seller that accepts the lowest payment as the opaque product provider (Dolan 2000). In this area, Fay (2009) studies the competition between Priceline.com and Hotwire.com. Both companies deal with opaque products, but the former uses a buyer-driven NYOP scheme while the latter uses a seller-driven, posted-price scheme. Chen et al. (2011) consider heterogeneous customers and compare the performance of posted-price and NYOP in the last minute opaque selling.

Among the existing works that study NYOP or opaque products, not much restriction has been put on the supply. For the limited number of papers that consider supply limit, it is usually assumed that there can be only one distribution channel. For example, in Gallego and Phillips (2004), opaque products are available only during regular seasons, while in Wang et al. (2009), Jerath et al. (2010), and Chen et al. (2011) opaque selling occurs only in last minutes. However, it should be noted that in relevant industries where NYOP usually applies — e.g., airlines, hotels, and rental cars — supply/time limits play the pivotal role on revenue earning (McGill and van Ryzin 1999), and that these products/services are normally available in multiple sales channels (Bitran and Caldentey 2003, Elmaghraby and Keskinocak 2003). In approximating the real world practice, we consider a general model where competing sellers can dynamically determine their channel strategy, pricing strategy, and inventory control strategy during a finite time horizon. To our best knowledge, very few paper incorporated all these factors thus far.

3. The Model

Consider two sellers (each of whom will be referred to as "he") with substitutable products, an intermediary NYOP firm (referred to as "it"), and a sequence of customers (each of whom will be referred to as "she"). A seller and his product have the same index i, where $i \in \{1,2\}$. Assume that seller i holds $\bar{x}_i$ units of initial inventory, which will expire after T periods and is not replenishable.

We use a backward time index $t = T, T-1, \ldots, 1$ to denote the current period, so a smaller number indicates that we are closer to the ending time. We refer to the time horizon from $t = T$ up to when the inventory is about to expire ($t = 1$) as the selling season of the products.

At the beginning of the selling season ($t = T$), sellers needs to determine their channel strategies. Each seller owns a direct channel (e.g., company website) and would contemplate whether to adopt the NYOP channel at the same time. We assume that the cost of revising channel strategy in season could be quite costly, thus the same strategy applies to all selling periods. Overall, there could be three kinds of channel structures: 1) Single-Channel (SC), where both sellers use direct channels only, 2) Dual-Channel (DC), where both sellers sell through both the direct and the NYOP channels, and 3) Semi-Dual-Channel (SDCi), where seller j uses direct channel only, and seller i adopts both the direct and the NYOP channels.

In each period $t \in \{T, T-1, ..., 1\}$, the same three-stage decision making takes place, as depicted in Figure 1.

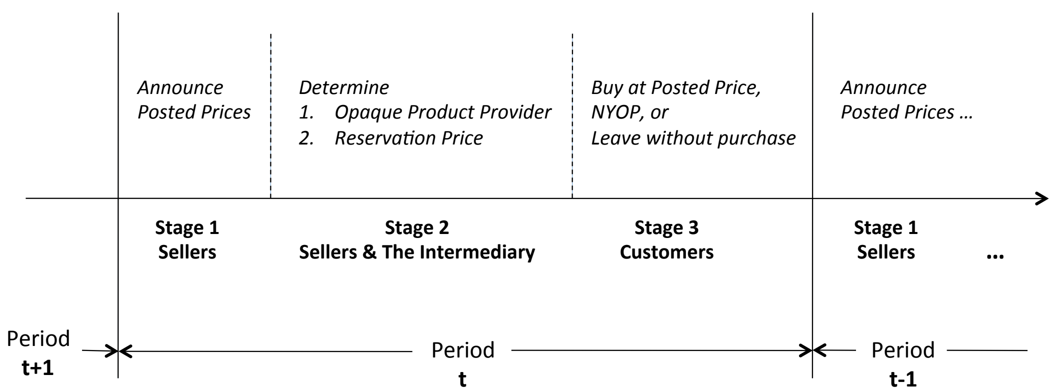

Figure 1 **Game sequence in each period.**

Stage 1: The sellers publish their posted prices, $\mathbf{p}_t = (p_{1t}, p_{2t})$, for purchases going through the direct channel during this period.

Stage 2: Each seller adopting the NYOP channel (e.g., seller i) sets a reservation price (e.g, r_{it}) with the intermediary firm. We model the intermediary firm as a revenue maximizer that benefit from the spread of NYOP bid and the reservation price (Dolan 2000). That is, if the intermediary firm pairs an NYOP bid b with seller i's product, the customer pays b, seller i receives r_{it} and the intermediary earns the spread $b - r_{it}$. Then under DC, the intermediary firm will find the lowest reservation price $r_t = \min \mathbf{r}_t = \min\{r_{1t}, r_{2t}\}$ and let seller $i = \arg\min_{k=1,2}\{r_{kt}\}$ be the opaque product provider. Specifically under DC, we assume that the intermediary firm has the discretion to share r_t with the sellers in facilitating competitive reservation price submission. Under SDC, where only seller i participates NYOP, the reservation price $r_t = r_{it}$ and the opaque product provider is i.

Stage 3: A customer arrives[3] and decides if she wants to buy directly from her preferred seller at the posted price, or name her own price b_t (which we will refer to as "make a bid" for the reminder of the paper) with the intermediary firm. In the second case, she may be matched up with either of the two sellers if the bid is accepted. In case the bid is rejected, the customer can always buy from the direct channel of her preferred seller. However, making a bid is considered a commitment to buy, and thus a customer cannot decline a product assigned by the intermediary firm if she discovers *ex-post* that being awarded the alternative product or buying at the posted price would make her better off. The customer is also not allowed to make a second bid with the NYOP channel after the first bid is rejected. These conform with the current practice of Priceline.com. [4]

Assumptions: We assume that the customer has valuation $\mathbf{v}_t = (v_{1t}, v_{2t})$ for the products and $v_{i,t}$'s are i.i.d. on $[0,1]$ with density $f_i(\cdot)$ and cumulative distribution function (cdf) $F_i(\cdot)$. The valuation is private information to the customer; the seller and the intermediary firm only know its distribution function, $F_{1,2}(\cdot)$. The customer can observe the channel structure and current posted prices ($\mathbf{p}_t$), but has no access to the reservation price (r_t) or who is the opaque product provider.

Many NYOP-relevant papers (e.g., Hann and Terwiesch 2003, Spann et al. 2004, Terwiesch et al. 2005 and Cai et al. 2009) assume that consumers adopt subjective priors in estimating the unobservable information such as reservation price. This practically reflects customers' generic expectation of the NYOP channel. [5] Conforming with this line of assumptions, we let the customers assume that: 1) the opaque product provider can be either seller 1 or seller 2 with probability α_1 and α_2 respectively, where $\alpha_1, \alpha_2 > 0$ and $\alpha_1 + \alpha_2 = 1$; and 2) the reservation price is a random variable $\hat{r}_t$ distributed on the support of $[0, \min\{p_{1t}, p_{2t}\}]$ with p.d.f. $g(\cdot)$ and c.d.f. $G(\cdot)$. Further, we assume that $\hat{r}_t$ has decreasing reversed hazard rate (DRHR), i.e., g/G is decreasing. This is equivalent to assuming that G is log-concave. Many (truncated) distributions satisfy this condition (e.g., uniform, normal, exponential, chi-squared, logit, etc.), hence this assumption is not too restrictive (Bagnoli and Bergstrom 2005).

Lastly, customers know about the availability status of a seller (e.g., a seller may show "in stock" or "stock out" in his direct channel) but not his real-time inventory level. For analytical

[3] This implicitly assumes that the each period is rather short. Similar assumptions have been widely applied in literature, e.g., Bhandari and Secomandi (2011), Kuo et al. (2011). The assumption that customer arrival rate equals to one can easily be relaxed to any fractional arrival rate by adjusting the distributions of valuations accordingly.

[4] As Dolan (2000) noted, "...only one offer was permitted in a seven-day period..." The policy may have been updated, but Priceline technically forbids repeated bidding within a limited among of period. Interested readers may refer to Fay (2004), Spann (2004), Terwiesch et al. (2005), and Hinz and Spann (2008) for problems involving repeated bidding/bid learning.

[5] The prior can be estimated through consumer research or survey. In §7, we demonstrate this via an experiment.

convenience, we further assume that the inventory levels $\mathbf{x}_t = (x_{1t}, x_{2t})$ are visible between the sellers themselves. Similar assumptions were used in many papers regarding competitive revenue management (e.g., Gallego and Hu 2013; Levin et al. 2009; Lin and Sibdari 2008) [6]. As our primary goal is to study how the presence of an NYOP channel may affect the strategic decisions of sellers, this assumption falls within the scope of the paper.

4. The Customers' Problem

We first analyze the problem in Stage 3, where customers make purchasing/bidding decisions given the channel options. Since the same problem repeats itself every period, in this section we omit the subscript t for ease of exposition.

Denote a customer's action by $a \in A = \{0, 1, 2, B\}$. In particular, $a = 0$ suggests that a consumer would not purchase from any of the three channels (two direct channels and one opaque channel), and $a = 1$ if a consumer would buy directly from seller 1 at the posted price p_1. Similar interpretation applies to $a = 2$. If one chooses to NYOP first, then $a = B$ and the customer will have to pick one from $A \backslash \{B\} = \{0, 1, 2\}$ if the bid is rejected.

We use $V_S^a(\mathbf{v})$ to represent the expected utility for a customer with valuation $\mathbf{v}$ taking action a under channel structure $S \in \{SC, SDC1, SDC2, DC\}$. For example, $V_S^0(\mathbf{v}) = 0$, $V_S^1(\mathbf{v}) = v_1 - p_1$ and $V_S^2(\mathbf{v}) = v_2 - p_2$ for any S. When $S = DC$, if a customer chooses to bid b first and buy from seller 2 if rejected, his expected utility is $G(b)(\alpha_1 v_1 + \alpha_2 v_2 - b) + \bar{G}(b)(v_2 - p_2)$, where $\bar{G}(\cdot) = 1 - G(\cdot)$. A customer would rationally choose the bid and exit option (direct channel 1, or 2, or empty-handed) in maximizing her expected utility. Thus, $V_{DC}^B(\mathbf{v}) = \max_{b \geq 0}\{G(b)(\alpha_1 v_1 + \alpha_2 v_2 - b) + \bar{G}(b) \max_{k=0,1,2} V_{DC}^k(\mathbf{v})\}$. Similar analysis applies to SDCi.

Finally, in characterizing the final purchase, let $\mathbf{H} = (u, c)$ be a product-price tuple where $u \in \{0, 1, 2, O\}$ denotes the purchased product (no purchase, product 1, 2, or the opaque product respectively) and c is the price that one has to pay for the corresponding product.

4.1. SC: when neither seller adopts the NYOP channel

When both sellers announce that they will not participate in the opaque channel but operate the direct channel only, apparently the action "B" is not available to the consumers. A customer's expected utilities under the rest of the actions are:

$$V_{SC}^a(\mathbf{v}) = \begin{cases} 0 & \text{if } a = 0 \\ v_a - p_a & \text{if } a = 1, 2 \end{cases}$$

[6] For imperfect information on parameters such as demand distributions or inventory levels, one may refer to Perakis and Sood (2006), Levina et al. (2009), Zhang and Kallesen (2008), etc.

Comparing the expected utilities, the best action $a^*(\mathbf{v}) = \arg\max_{a\in\{0,1,2\}}\{V^a(\mathbf{v})\}$ is then

$$a^*_{SC}(\mathbf{v}) = \begin{cases} 0 & \text{if } \max\{v_1 - p_1, v_2 - p_2\} < 0; \\ 1 & \text{if } v_1 - p_1 \geq \max\{v_2 - p_2, 0\} \text{ and } p_1 \leq v_1; \\ 2 & \text{if } v_2 - p_2 \geq \max\{v_1 - p_1, 0\} \text{ and } p_2 \leq v_2; \end{cases}$$

and the final purchasing realization is

$$\mathbf{H}_{SC}(\mathbf{v}) = \begin{cases} (0,0) & \text{if } \max\{v_1 - p_1, v_2 - p_2\} < 0; \\ (1,p_1) & \text{if } v_1 - p_1 \geq \max\{v_2 - p_2, 0\}; \\ (2,p_2) & \text{if } v_2 - p_2 \geq \max\{v_1 - p_1, 0\}. \end{cases}$$

4.2. SDC: When only seller i adopts the opaque channel

Assume that seller i participates in the NYOP channel and seller j announces that his product is available through the direct channel only. Customer's expected utility can be expressed as:

$$V^a_{SDCi}(\mathbf{v}) = \begin{cases} 0 & \text{for } a = 0 \\ v_a - p_a & \text{for } a = 1,2 \\ \max_{b\geq 0}\ G(b)(v_i - b) + \bar{G}(b)\max_{k=0,1,2}\{V^k_{SDCi}(\mathbf{v})\} & \text{for } a = B. \end{cases}$$

The best action $a^*(\mathbf{v}) = \arg\max_{a\in\{0,1,2,B\}}\{V^a(\mathbf{v})\}$ can be identified as

$$a^*_{SDCi}(\mathbf{v}) = \begin{cases} j & \text{if } v_j - p_j \geq \max\{v_i - p_i, 0\} \text{ and } p_j < v_j - v_i; \\ B & \text{otherwise.} \end{cases}$$

For those who will bid first ($a^* = B$), the optimal bid b^* satisfies $b^* + \dfrac{G(b^*)}{g(b^*)} = v_i - \max\limits_{k=0,1,2}\{V^k(\mathbf{v})\}$. Since G/g increases in b, there exists a unique bidding level b^* that satisfies this condition. The final purchasing realization is then:

$$\mathbf{H}_{SDCi}(\mathbf{v}) = \begin{cases} (0,0) & \text{if } \max\{v_1 - p_1, v_2 - p_2\} < 0 \text{ and } r + \dfrac{G(r)}{g(r)} > v_i; \\ (i,p_i) & \text{if } v_i - p_i \geq \max\{v_j - p_j, 0\} \text{ and } p_i - r - \dfrac{G(r)}{g(r)} < 0; \\ (j,p_j) & \text{if } v_j - p_j \geq \max\{v_i - p_i, 0\} \text{ and } p_j - r - \dfrac{G(r)}{g(r)} < v_j - v_i; \\ (O,b^*) & \text{otherwise.} \end{cases} \quad (1)$$

4.3. DC: When both sellers adopt the NYOP channel

In the case where both sellers sell through dual channels, the expected utility for a customer can be characterized as follows:

$$V^a_{DC}(\mathbf{v}) = \begin{cases} 0 & \text{for } a = 0 \\ v_a - p_a & \text{for } a = 1,2 \\ \max_{b\geq 0}\ G(b)(\alpha_1 v_1 + \alpha_2 v_2 - b) + \bar{G}(b)\max_{k=0,1,2}\{V^k_{DC}(\mathbf{v})\} & \text{for } a = B. \end{cases}$$

The best action for a customer with valuation $\mathbf{v}$ can be obtained by $a^*(\mathbf{v}) = \arg\max_{a\in\{0,1,2,B\}}\{V^a(\mathbf{v})\}$. In particular,

$$a^*_{DC}(\mathbf{v}) = \begin{cases} 1 & \text{if } v_1 - p_1 \geq \max\{v_2 - p_2, 0\} \text{ and } p_1/\alpha_2 < v_1 - v_2; \\ 2 & \text{if } v_2 - p_2 \geq \max\{v_1 - p_1, 0\} \text{ and } p_2/\alpha_1 < v_2 - v_1; \\ B & \text{if } \max\{v_1 - p_1, v_2 - p_2\} < 0; \\ & \text{or, } v_1 - p_1 \geq \max\{v_2 - p_2, 0\} \text{ and } p_1/\alpha_2 \geq v_1 - v_2; \\ & \text{or, } v_2 - p_2 \geq \max\{v_1 - p_1, 0\} \text{ and } p_2/\alpha_1 \geq v_2 - v_1. \end{cases}$$

That is, customers who can afford and strongly prefer product i to product j, where $i, j \in \{1, 2\}$ and $i \neq j$, will buy product i directly at posted price p_i. Those that will NYOP first contain three kinds of customers: budget customers who can afford neither product at the posted prices, and customers who can afford and prefer product i to product j in a less-than-strong manner. Among these three kinds of customers, a rejected bid will lead to direct channel purchase for the latter two kinds of customers, and a budget customer will always leave empty handed if her bid is not accepted.

Proposition 1. *When both sellers sell through the opaque channel, 1) customers with limited degree of differentiation between the two products and 2) customers who can afford neither product at listed prices will NYOP first. In particular, the bid decreases with the degree of differentiation ($|v_i - v_j + p_j - p_i|$) for the first set of customers, and increases with the expected valuation ($\alpha_i v_i + \alpha_j v_j$) for the second set of customers.*

The optimal bid b^* for those who will NYOP first ($a^* = B$) can be achieved by the first-order-condition of V^B: $b^* + \dfrac{G(b^*)}{g(b^*)} = \alpha_1 v_1 + \alpha_2 v_2 - \max_{k=0,1,2}\{V^k(\mathbf{v})\}$. For the same reason stated in §4.2, there exists a unique bidding level b^* that satisfies this equation. As $b + \dfrac{G(b)}{g(b)}$ increases in b, customers whose valuation $\mathbf{v}$ satisfies $r + \dfrac{G(r)}{g(r)} > \alpha_1 v_1 + \alpha_2 v_2 - \max_{k=0,1,2}\{V^k(\mathbf{v})\}$ will bid lower than the reservation price ($b^*(\mathbf{v}) < r$), and hence be rejected in the NYOP channel. Consequently, the ultimate purchasing outcome can be characterized by

$$\mathbf{H}_{DC}(\mathbf{v}) = \begin{cases} (0,0) & \text{if } \max\{v_1 - p_1, v_2 - p_2\} < 0 \text{ and } r + \dfrac{G(r)}{g(r)} > \alpha_1 v_1 + \alpha_2 v_2; \\ (1, p_1) & \text{if } v_1 - p_1 \geq \max\{v_2 - p_2, 0\} \text{ and } p_1 - r - \dfrac{G(r)}{g(r)} < \alpha_2(v_1 - v_2); \\ (2, p_2) & \text{if } v_2 - p_2 \geq \max\{v_1 - p_1, 0\} \text{ and } p_2 - r - \dfrac{G(r)}{g(r)} < \alpha_1(v_2 - v_1); \\ (O, b^*) & \text{otherwise.} \end{cases}$$

First of all, budget customers with low expected valuation of the opaque product will bid low and will not be awarded any product. Then, customers with strong preference for product 1 over product 2 will buy directly instead of bidding, and those with moderate preference will present insufficient bids and turn back to direct channel 1 in the end. These represent the customers that will ultimately buy from direct channel 1. Similar analysis applies to direct channel 2. Lastly, those who successfully obtain a product via NYOP consist of 1) budget customers with relatively high valuation of the two products, and 2) other customers who can afford some product at posted price, and have weak preference between the two direct channels.

5. Dynamic Pricing of the Sellers

This section provides structural results on the pricing decisions of two competing sellers (Stage 1 and Stage 2). The problem will be addressed in two steps. First, given posted prices $\mathbf{p}_t$ in period t, what reservation price r_t will the sellers end up with and who will be the opaque provider? This corresponds to the Stage 2 decision. Second, we study whether equilibrium posted prices can be achieved — Stage 1 decision making.

For tractability, we assume that the customers' valuation $\mathbf{v}$ are uniformly distributed on $[0,1]\times[0,1]$. Also, customers assumes equal probability for either seller to be the opaque product provider ($\alpha_1=\alpha_2=0.5$) and uniform distribution for the reservation price, i.e., $\hat{r}_t\sim U[0,\min\{p_{1t},p_{2t}\}]$. All results in this section are derived based on Assumption 1 [7]. The robustness and rationality of this assumption will be discussed in §7.1.

Assumption 1. *For any $t\in\{T,T-1,...,1\}$, customers assume that $\mathbf{v}_t\sim U[0,1]\times[0,1]$, $\alpha_1=\alpha_2=0.5$ and $\hat{r}_t\sim U[0,p_t]$, where $p_t=\min\{p_{1t},p_{2t}\}$ is the minimum listed price in period t.*

For the rest of the paper, denote by $\mathbf{p}^*(\mathbf{x}_t,t)$ the equilibrium posted price in period t when the inventory levels are $\mathbf{x}_t=(x_{1t},x_{2t})$, and $\Pi_i(\mathbf{x}_t,t)$ the expected revenue for seller i given that posted prices will be set in the equilibrium manner in all future periods.

5.1. Monopolistic seller

We first analyze the benchmark case when there is only one seller, say, seller 1, in the market. This practically happens when all inventories at seller 2 are sold out. Since availability is known to the customers, in this instance the NYOP channel loses its opaqueness and customers will expect that only product 1 will be awarded for successful bids. We briefly state the key findings as follows.

Theorem 1. *A monopolistic seller maximizes his expected revenue by using only the direct channel.*

The result shows that the value of NYOP channel is minimum in absence of competition and opaqueness, for missing the shielding benefit. In this circumstance, the NYOP channel allows

[7] Numerical simulation suggests that similar results hold under more general situations.

discount request at little cost, as one can always buy from the direct channel guaranteeing the same product if her bid is turned down. Therefore, it is optimal for the customers to participate in the NYOP roulette in the first place, and the direct channel in the second. Taking this into account, it is to the benefit of the seller to sell everything via the direct channel only. Specifically, we can characterize the optimal direct-channel pricing decision as follows:

$$p_{1t} = \begin{cases} \frac{1}{2} & \text{when } t = 1; \\ \frac{1 + \Pi_1(x_1, t-1) - \Pi_1(x_1 - 1, t-1)}{2} & \text{when } t > 1, \end{cases}$$

where $\Pi_1(x_1, t)$ is the optimal expected profit for the monopolist seller in period t with x_1 units of inventory on hand:

$$\Pi_1(x,t) = \begin{cases} \frac{1}{4} & \text{when } t = 1; \\ \Pi_1(x_1 - 1, t-1) + \left[\frac{1 + \Pi_1(x_1, t-1) - \Pi_1(x_1 - 1, t-1)}{2}\right]^2 & \text{when } t > 1. \end{cases}$$

5.2. SDC: Competing sellers with only one adopting the NYOP channel

This subsection considers equilibrium pricing under the SDC structure, where only one seller joins the NYOP channel throughout the selling season. Without loss of generality, let this be seller 1. That is, customers can estimate that product 1 will always be awarded for the winning bids. With this in mind, seller 1 has to properly determine his posted and reservation prices, p_{1t} and r_{1t}, to control the sales flow.

Given any reservation and the posted prices $(\mathbf{r}, \mathbf{p})$, let $\Omega_k(\mathbf{r}, \mathbf{p})$, where $k = 0, 1, 2, B$, be the fraction of customers who will *ultimately* buy nothing, product 1 from seller 1, product 2 from seller 2, or the opaque product from the intermediary firm, respectively. In another word, $\Omega_k(\mathbf{r}, \mathbf{p})$ is the probability that a customer's $\mathbf{H}_{SDC1} = k$. Then, at $(\mathbf{r}_t, \mathbf{p}_t)$,

$$\begin{aligned} \Pi_1(x_1, x_2, t) = {} & \Omega_O(\mathbf{r}_t, \mathbf{p}_t)\left[r_{1t} + \Pi_1(x_1 - 1, x_1, t-1)\right] + \Omega_1(\mathbf{r}_t, \mathbf{p}_t)\left[p_{1t} + \Pi_1(x_1 - 1, x_2, t-1)\right] \\ & + \Omega_2(\mathbf{r}_t, \mathbf{p}_t)\Pi_1(x_1, x_2 - 1, t-1) + \Omega_0(\mathbf{r}_t, \mathbf{p}_t)\Pi_1(x_1, x_2, t-1) \\ \Pi_2(x_1, x_2, t) = {} & \left[\Omega_O(\mathbf{r}_t, \mathbf{p}_t) + \Omega_1(\mathbf{r}_t, \mathbf{p}_t)\right]\Pi_2(x_1 - 1, x_2, t-1) \\ & + \Omega_2(\mathbf{r}_t, \mathbf{p}_t)\left[p_{2t} + \Pi_2(x_1, x_2 - 1, t-1)\right] + \Omega_0(\mathbf{r}_t, \mathbf{p}_t)\Pi_2(x_1, x_2, t-1). \end{aligned}$$

Back in §4.2, (1) suggests that if the reservation price is high, i.e., $2r_{1t} \geq p_{1t}$, then no purchase will be done through the NYOP channel and all sales will be realized via direct channels ($\mathbf{H}(\mathbf{v})_{SDC1} \in \{1, 2\}$); otherwise, customers will buy product 1 via the opaque channel and product 2 through the direct channel ($\mathbf{H}(\mathbf{v})_{SDC1} \in \{2, O\}$). Essentially, seller 1 in each period can only sell through the NYOP or the direct channel. Then, given p_{2t}, the best pricing reaction for seller 1 is

$$(p^*_{1t}, r^*_{1t}) = \begin{cases} (arg\max_{0 < p_{1t} < 1} \Pi_1^D, 1) & \text{when } \Pi_1^D > \Pi_1^O; \\ (1, arg\max_{0 < r_{1t} < 1} \Pi_1^O) & \text{when } \Pi_1^D \leq \Pi_1^O, \end{cases}$$

where $\Pi_1^D = \Pi_1|_{r_{1t}=r_{2t}=1}$ and $\Pi_1^O = \Pi_1|_{p_{1t}=r_{2t}=1}$. Given (p_{1t}, r_{1t}), the optimal reaction for seller 2 can easily be presented as $p_{2t}^* = arg\max_{0<p_{2t}<1} \Pi_2|_{r_{2t}=1}$.

THEOREM 2. *Under SDC1, in any period t there exists a pure-strategy Nash equilibrium in reservation and posted prices $(r_{1t}^*, p_{1t}^*, p_{2t}^*)$.*

5.3. DC: Competing sellers both adopting the NYOP channel

We now study the DC scenario in which both sellers adopt the NYOP channel, thus its opaqueness is retained. We further define the *marginal value of inventory* at period t for each seller as:

$$\widetilde{\Pi_1}(\mathbf{x}, t) = \Pi_1(x_1, x_2 - 1, t) - \Pi_1(x_1 - 1, x_2, t)$$
$$\widetilde{\Pi_2}(\mathbf{x}, t) = \Pi_2(x_1 - 1, x_2, t) - \Pi_2(x_1, x_2 - 1, t).$$

$\widetilde{\Pi_i}$ measures the difference in seller i's expected revenue if he lets a customer go to his rival instead of serving the customer himself. In essence, it is the cost for seller i to serve a customer at period t and inventory level $\mathbf{x}$. Since the intermediary can disclose r_t to the sellers, it can be shown that in equilibrium the submitted reservation prices (r_{1t}^*, r_{2t}^*) reflect the marginal value of inventory for each seller, and the reservation price of the opaque product is the lower of the two.

PROPOSITION 2. *Under DC, at any period t,*

(i). seller with lower marginal value of inventory will be the opaque provider; [8]

(ii). the reservation price represents the higher marginal value of inventory between the two sellers, i.e., $r^*(\mathbf{x}, t) = \max_{i=1,2} \widetilde{\Pi_i}(\mathbf{x}, t-1)$.

Given Stage 2 decision (reservation price and opaque product provider), we can now verify the existence of pure-strategy Nash equilibrium (NE), $p_1^*(\mathbf{x}, t), p_2^*(\mathbf{x}, t)$, for Stage 1. In particular, we need to check if the expected profit function is quasi-concave in p_{it}. While this can be analytically proved for uniform distribution, i.e., $\mathbf{v}_t \sim [0,1] \times [0,1]$, numerical experiment suggests that NE exists in general instances, e.g., normal distribution.

THEOREM 3. *Under DC at any time epoch t, there exists a pure-strategy Nash equilibrium in posted prices $\mathbf{p}_t^*$.*

Due to the generality of the model, Theorem 3 only provides structural results to the Stage 1 problem. However, the existence of NE under Assumption 1 allows us to conduct some numerical studies in the next section, which generate more insights on the expected profits, equilibrium prices, as well as equilibrium channel strategies. Also, additional analysis of the expected profit helps us obtain more insights of the reservation price.

[8] In case of a tie, each serves as the opaque provider with an equal probability.

PROPOSITION 3.

(i) In the last period $t = 1$, the reservation price is zero if both sellers are in stock.

(ii) At period $t > 1$, reservation price r^* is zero if both sellers oversupply, i.e., $x_{i,t} \geq t$ for $i = 1, 2$.

The main implication is that sellers may offer last-minute sales ($t = 1$) that lead to arbitrary bids at the NYOP channel ($r_t^* = 0$). [9] Similar NYOP channel discounts can also take place throughout the selling seasons when supply dominates demand. Note that in the latter case, deep discounts are mainly driven by competition and the existence of NYOP platform. Otherwise, the discounts are less dramatic. In monopolistic market (§5.1), the seller will merely abandon the excess capacity rather than giving away deep discount in posted price. In a competitive market without NYOP channel (as we will see later in §6), oversupply may decrease the posted prices but not to the extent that customers can possibly pay minimum for the product ($r_t^* = 0$).

6. Numerical Analysis: Pricing, Profit and Channel Strategy

The model thus far identifies the dynamic processes in solving equilibrium prices and profits under each channel structure. However, closed form solutions to these problems are generally not tractable. In this section, we conduct numerical study to provide an outlook of these solutions. Specifically, in §6.1, we compare the equilibrium posted and reservation prices and expected profit for each seller under the three channel structures: SC, DC and SDC. In §6.2, we further allow the sellers to choose their channel strategies, suggesting some key drivers for NYOP adoption. All analysis is conducted based on Assumption 1.

6.1. Comparison of SD, DC and SDC

6.1.1. Expected Profit Assuming the remaining sales time is $t = 10$, Figure 2 illustrates the expected profit for a particular seller (e.g., seller 1) given the inventory levels at his and his competitor's sites. The top-left, top-right, and bottom graphs depict profits under DC, SC and SDC structures, respectively.

This figure has several interesting implications. First of all, under SDC structure, the seller that adopts dual-channel strategy overall gains higher expected profit than the one with single-channel strategy. This can be obtained by comparing the lower two graphs in Figure 2. Thus, the single-channel strategy identified in Theorem 1 can be entirely reversed in the presence of competition, that it is best for a seller to use both NYOP and direct channels if his rival only adopts the latter.

Second, the expected profit is lower in DC than in SC, as suggested by the top two graphs. That is, the sellers are worse off under widely-adopted NYOP, and the gap in profit is more evident as

[9] The customers may factor this into account when deciding their bid at $t = 1$. In our model, this is the only moment in which customers do not need the prior as assumed in Assumption 1.

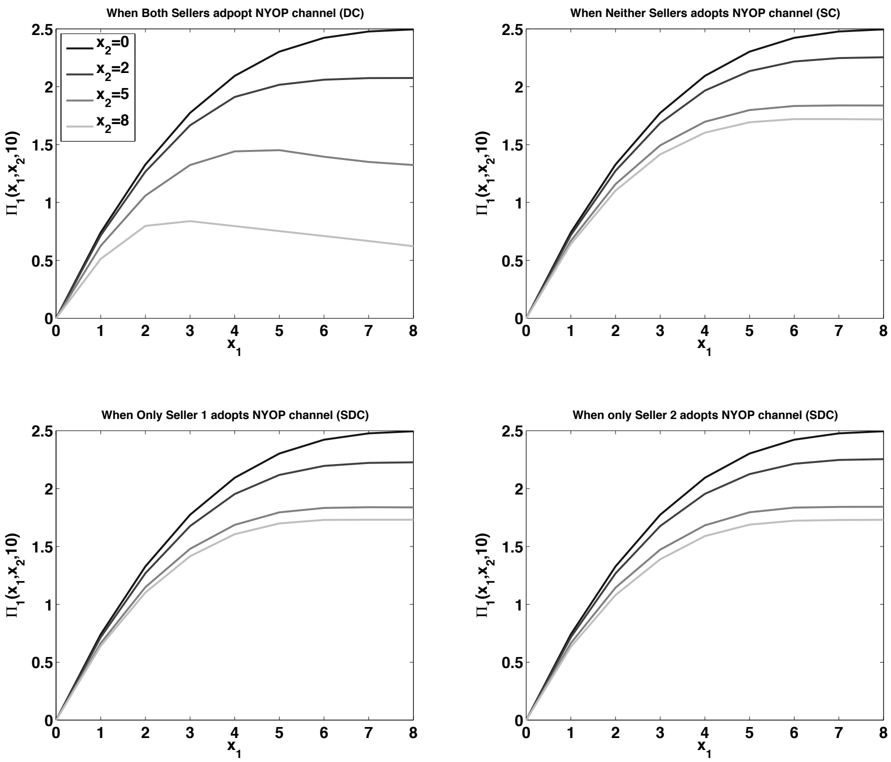

Figure 2 Expected profit at given time epoch $t = 10$

the competition becomes more intense (e.g., higher x_1 or x_2). In addition, while one's expected profit generally increases with his own inventory level under SC and SDC, this does not hold for DC. For example, when seller 2 has 8 units on hand ($x_2 = 8$), the expected profit of seller 1 decreases in his own inventory level when $x_1 \geq 3$. This could possibly be due to two reasons. First, the NYOP channel lowers down realized sales revenue. Note that NYOP provides a platform on which sellers compete to sell at a low price without damaging the integrity of their direct channel. As competition becomes more intense, the reservation price at the NYOP channel could become rather low (eventually becoming $r = 0$ in case of over-supply). Thus, there is a reasonable chance that some product quantity will be sold at lower prices in DC than SC. Second, the NYOP channel intensifies the listed-price competition, as will be discussed next.

6.1.2. Equilibrium Posted Price Figure 3 illustrates the equilibrium posted price under DC (left column) and SC (right column) respectively. Apparently, prices are lower under DC than SC, reflecting the intense price competition after the NYOP channel. Indeed, in the absence of the NYOP channel, sellers compete publicly through posted prices only, and there is less uncertainty about whose product the customer will eventually buy. When the NYOP channel is adopted by both sellers, each is aware that the customers that he used to serve can now be "stolen" by his rival via NYOP, or even cannibalized by his own part of opaque product. It is then not surprising that posted price falls in equilibrium.

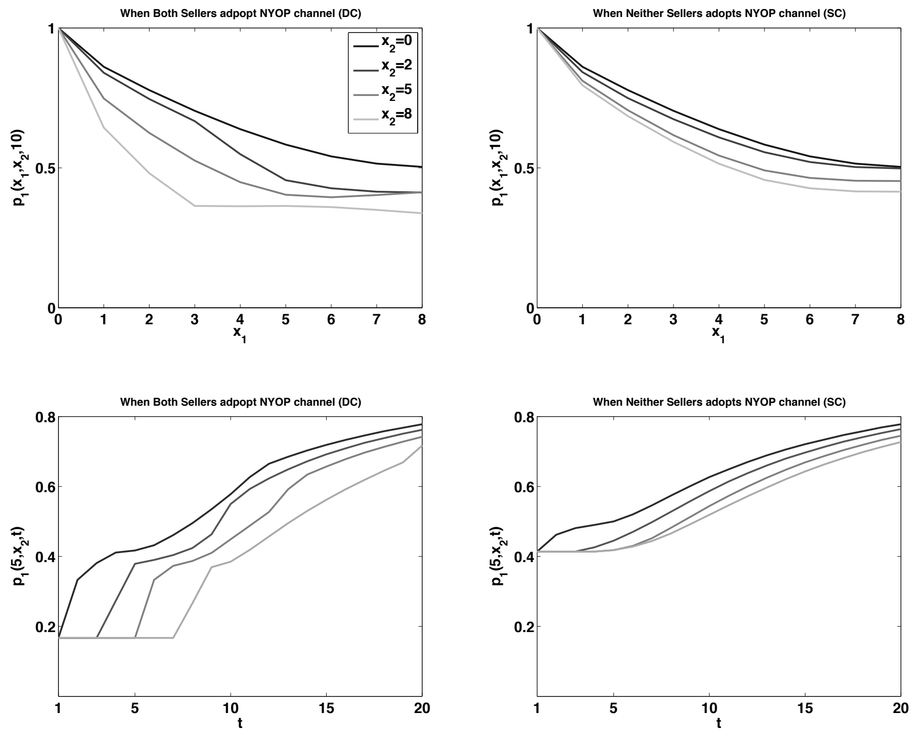

Figure 3 Equilibrium posted price at given time epoch $t = 10$ (top graphs) or inventory level $x_1 = 4$ (lower graphs).

Also in Figure 3, the top graphs depicts how the posted price varies with one's own inventory level (x_1), and the lower graphs demonstrates how equilibrium posted price drops with the sales horizon (t). In both cases, it can be observed that prices in DC are more sensitive to inventory level changes and sales timing than in SC.

6.1.3. Reservation Price Figure 4 illustrates the reservation price r when it applies, i.e., under DC and SDC. The contrast between structures is more evident for reservation price. Overall, r is higher under SDC, indicating that less discount will be offered when the opaque channel is subject to monopolistic decision power. For the same reason, the reservation price is much less sensitive to time or inventory level under SDC than DC. Take $x_1 = 4$, $x_2 = 8$ for example: throughout the entire selling season from $t = 1 \sim 20$, the reservation price in SDC only drops from 0.3 to 0.21, while in DC the reservation price can be from as high as 0.46 down to 0. From the seller's perspective, in SDC, the NYOP channel acts as a static discount channel whose main driver is to expand the market.

6.2. Equilibrium Channel Decisions

As Figure 2 illustrates, sellers are worse off in presence of NYOP. Then a natural question arise: if the NYOP channel is not beneficial, why would the sellers ever adopt them? We explore insights to this question in what follows.

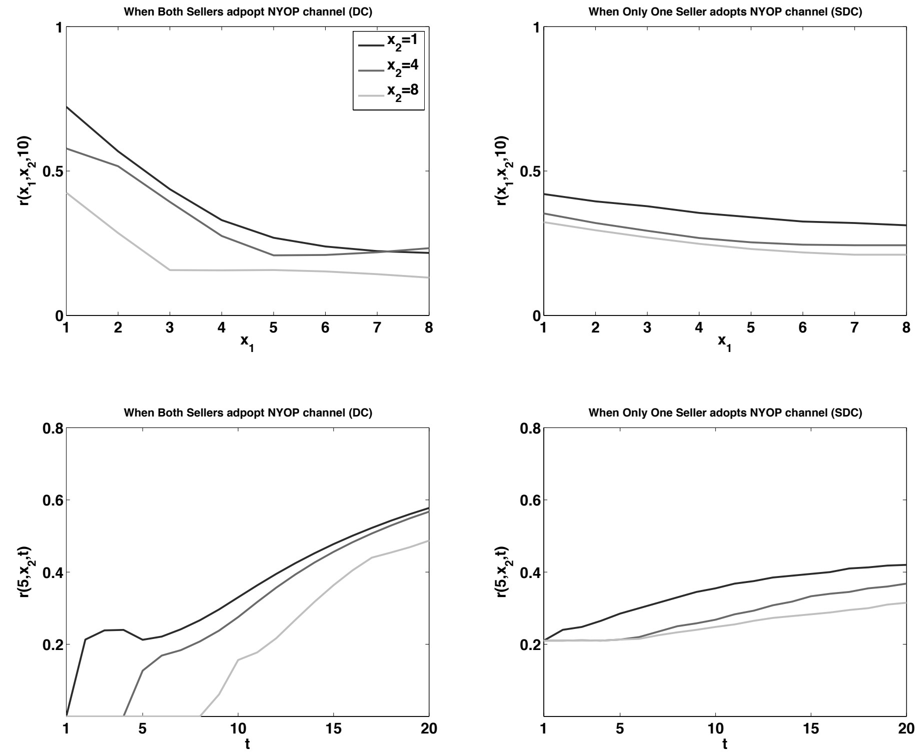

Figure 4 **Reservation price at given time epoch $t = 10$ (top graphs) or inventory level $x_1 = 4$ (lower graphs).**

Assume that the sellers need to determine their channel strategy (whether to participate in NYOP for the entire selling season) at the beginning of the game. Denote $S_i = 1$ if seller i decides to adopt NYOP and $S_i = 0$ otherwise. Thus $(S_1, S_2) = (0, 0)$ results in the SC structure, $(1, 1)$ the DC structure, $(1, 0)$ the SDC1 and $(0, 1)$ the SDC2. In addition, assume that S_1 and S_2 are publicly known (possibly through the NYOP site's own advertisement, or seller's announcement). Then, customers are informed about the underlying opaque product(s) to start with. For example, if $(S_1, S_2) = (1, 0)$, customers will know for certain that any opaque product will turn out to be product 1. On the other hand, under DC (i.e., $S_1 = S_2 = 1$) there is no way to tell which product will be awarded ahead of time. In this case, the customer will rely on the prior in estimating the opaque product realization, and its reservation price.

Suppose that the sales season is composed of t periods. Given channel strategies of the two parties, seller i can assess his expected profit at the current moment as $\Pi_i^{S_1 S_2}(\mathbf{x}, t)$. Then, *SC* will be the equilibrium channel strategy if $\Pi_1^{00}(\mathbf{x}, t) \geq \Pi_1^{10}(\mathbf{x}, t)$ and $\Pi_2^{00}(\mathbf{x}, t) \geq \Pi_2^{01}(\mathbf{x}, t)$; *DC* will be the equilibrium channel strategy if $\Pi_1^{11}(\mathbf{x}, t) \geq \Pi_1^{01}(\mathbf{x}, t)$ and $\Pi_2^{11}(\mathbf{x}, t) \geq \Pi_2^{10}(\mathbf{x}, t)$; *SDC* with only seller 1 participates NYOP is the equilibrium channel strategy if $\Pi_1^{10}(\mathbf{x}, t) \geq \Pi_1^{00}(\mathbf{x}, t)$ and $\Pi_2^{10}(\mathbf{x}, t) \geq \Pi_2^{11}(\mathbf{x}, t)$; and *SDC* where only seller 2 participates NYOP will be the equilibrium channel strategy if $\Pi_1^{01}(\mathbf{x}, t) \geq \Pi_1^{11}(\mathbf{x}, t)$ and $\Pi_2^{01}(\mathbf{x}, t) \geq \Pi_2^{00}(\mathbf{x}, t)$.

We pick four time lengths $t = 18, 12, 5, 1$, in approximating long versus short selling seasons,

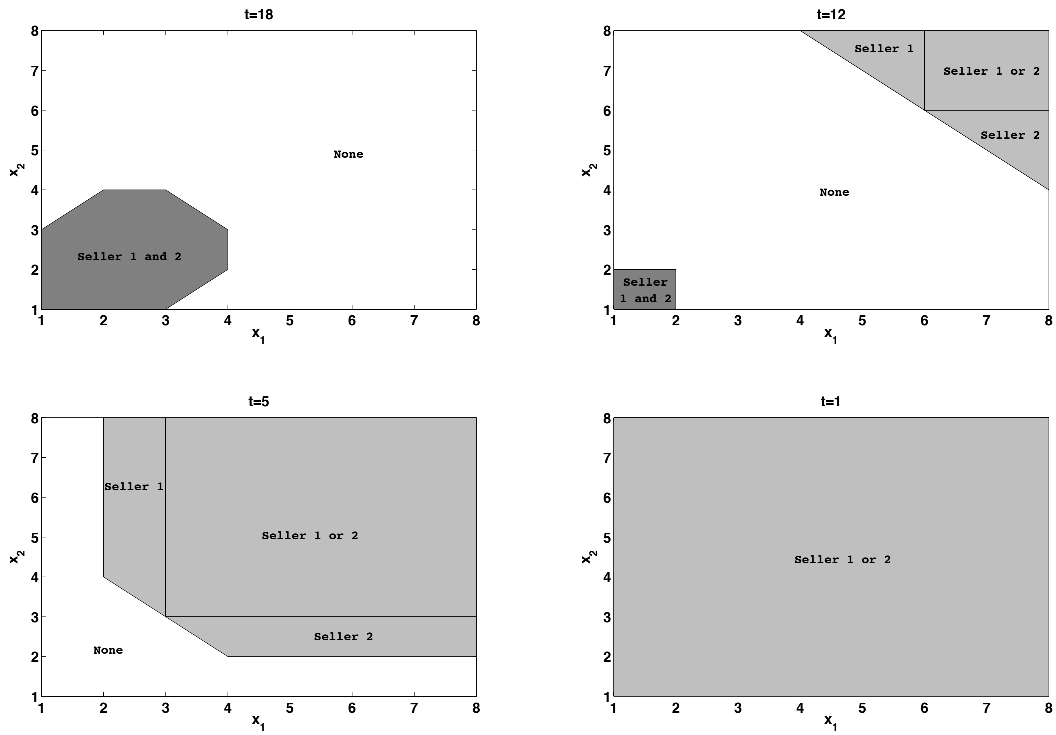

Figure 5 **Equilibrium channel choice: who will sell through the NYOP channel?**

and depict the equilibrium channel strategy in Figure 5. One can observe that the equilibrium heavily depends upon the initial inventory levels and the length of selling horizon. For example, when the time line is rather short ($t = 1$), sellers will differentiate themselves via channel strategies. Specifically, only one of them will adopt dual channels and the other will sell through direct channel only. The same holds true when the time line is moderately short ($t = 5$), except that single channel strategy may become the NE if supply is scarce for at least one seller. Therefore, even though the marginal value of the inventory is low at the last minute, sellers will still be able to avoid the ferocious pricing war by proper channel differentiation. However, as the graphs with $t = 18$ and $t = 12$ suggest, such channel differentiation vanishes with lengthened selling horizon. Indeed, with moderate inventory level and sufficient demand yet to come, a seller finds it less pressured to sell his products via a discounted channel if his rival does not.

The area that attracts most of our interest is when DC becomes the equilibrium, i.e., both seller 1 and 2 will adopt NYOP channel. We would like to note that in the darkest area of Figure 5, both DC and SC (that neither seller will adopt NYOP) are equilibriums. That is to say, when the selling horizon is long and inventory level at both sellers are low, it is possible for both sellers to adopt NYOP and direct channels at the same time. Since this is not a last minute sale ($t >> 0$), the reservation price can be reasonably high. Hence, a seller may find it better to compete on the opaque platform than losing customers to a dual-channel rival.

In general, we find that all three kinds of channel structures can appear in equilibrium depending on the timing and inventory levels. Sellers can properly use NYOP channel in differentiating themselves during last minutes sale, or guarding their market ground at the beginning of the selling season. In either case, zero reservation price (or free give-away) can be blocked by rational channel choice between the sellers.

7. Discussion and Extensions

Businesses today are under increasing pressure to refine their distribution channel and pricing strategies. In this study, we aim at understanding how the innovation of NYOP opaque channel may impact the industry with direct sales channels. We do this by developing a model framework that accommodates a full line of stakeholders, including the upstream sellers, the NYOP intermediary firm, and the downstream customers. The model allows explicit analysis on the optimal or equilibrium decision, and provides important managerial implications for each stakeholder. We have also illustrated how limited inventory levels and selling horizon may affect sellers' channel strategies. To the best of our knowledge, this is one of the first papers that consider operations issues for the NYOP-opaque-channel problem.

While our paper deals with a stylized model, some assumptions can be validated, and there are many directions in which this work can be extended. We discuss a few of them below.

7.1. Subjective Prior in the NYOP Channel

The equilibrium analysis and numerical studies are all based on Assumption 1, according to which: (1) customers assign equal probability to the opaque product 1 and product 2 (i.e., $\alpha_1 = \alpha_2 = 0.5$); and (2) there is equal probability of price being any number in the interval $[0, p_t]$, where p_t is the minimum posted price (i.e., $\hat{r}_t \sim U[0, p_t]$). [10]

Experiment Design. In order to verify the assumption about the customers' belief an experiment was designed and conducted at a Canadian university. The participants were students who had taken the entry-level supply chain or probability courses and thus had background knowledge of the experiments context. The participation was voluntary and participants received cash incentive comprising of a fixed participation fee and a performance-based bonus.

The experiment was conducted on a one-to-one basis between every participant and the experimenter. The experimenter explained to the subject the experimental context: the participant needs to book a 3-star hotel room in a particular region, and choose one of the following three options: 1) book Hotel A and pay \$80 upfront, 2) book Hotel B and pay \$90 upfront, or 3) submit a bid of one of the six values (\$40, \$50, \$60, \$70, \$80, or \$85). If the third option is chosen, then the

[10] Note that we implicitly assume that seller's cost per room is zero. In presence of any basic maintenance cost, the accepted price (r_t or p_t) can be considered as the margin that a seller charges on top of the cost.

bid may be accepted and either a room in Hotel A or Hotel B is awarded. If, however, the bid is rejected, then the participant has to decide between the first two options.

After explanation of the decision process, the participants were given their monetary valuations of the hotels and the formulae for the performance-based reward calculation. This reward value depended on the achieved utility level, i.e.,"valuation – price to pay."

Following the explanation, each participant would be given three scenarios:

- Scenario 1: Neither information about the probability for Hotel A or B to appear in the end nor the probability for the bid acceptance is given. This scenario closely approximates the decision environment at Priceline.

- Scenario 2: The probability for each hotel to appear is given, e.g., $(\alpha_1, \alpha_2) = (0.5, 0.5)$, but information about the minimum acceptance price is withheld.

- Scenario 3: Both the probability for each hotel to appear (it is the same as in Scenario 2) as well as the probability of accepting the minimum price (i.e., price \$40, \$50, \$60, \$70, \$80 would be accepted with probability $20\%, 40\%, 60\%, 80\%, 100\%$, respectively) are given.

Observe that in Scenario 3, the participants are given priors which correspond to priors stated in Assumption 1. However, in order to make the context simple yet realistic, discrete uniform distribution and a basic maintenance cost of \$40 are used.

The participants are given the first scenario and asked to decide on one of the three options. If they selected option 3 then they had to choose the bidding amount. If their bid was rejected, then they had to choose the first or the second option. Then, they moved to the next round and were given the second scenario and in the last round they were given the third scenario.

After a participant completed the experiment, the experimenter determined the reward for this participant. Note that the participant neither knew their hotel assignment nor the reward level following their activity in each round until the end of the experiment. Hence, the experiment minimized the learning effect.

Experiment Results. We tested four sets of parameters with valuations $(v_1, v_2) = (100, 130)$, or $(80, 90)$, and $(\alpha_1, \alpha_2) = (0.5, 0.5)$ or $(0.2, 0.8)$. Results are summarized in Table 2. Although the actual biddings do not conform with model predictions precisely (participants tend to overbid when valuations are high, i.e., $(v_1, v_2) = (100, 130)$), certain qualitative consistencies between the actual and predicted bids can be observed.

First, the results confirm the impact of valuation on bidding as predicted by the model. For example, when $\alpha_1 = \alpha_2 = 0.5$, the model suggests that customers with valuation $(v_1, v_2) = (100, 130)$ will bid at 50 and those with $(v_1, v_2) = (80, 90)$ will increase their bids to 60. Experimental results for all three scenarios confirm this effect.

(v_1, v_2)	(α_1, α_2)	Round 1 (No Info)	Round 2 (Half-Info)	Round 3 (Full Info)	H_0: *Impact of full info*
(100,130)	(0.5, 0.5)	60.83 (9.96)	58.33 (10.3)	[50] 60.83** (5.15)	p=1.00
(80, 90)	(0.5, 0.5)	62.5 (9.62)	59.55 (13.5)	[60] 62.5 (7.54)	p=1.00
(100,130)	(0.2, 0.8)	58.42 (16.4)	68.33 (13.87)	[60] 67.08* (10.54)	p=0.1379
(80, 90)	(0.2, 0.8)	60 (7.39)	62.5 (7.54)	[60] 63.33 (7.78)	p=0.2936

$^{*}p < 0.1, ^{**}p < 0.01$

H_0: *Bidding amount equals to model prediction b^* (the value in [ ])*

Table 2 Average bid (standard deviation) in the experimental survey.

Second, the results confirm the impact of prior on bidding. Consider $(v_1, v_2) = (100, 130)$ for example. Our model suggests that when (α_1, α_2) is changed from $(0.5, 0.5)$ to $(0.2, 0.8)$, the bid will be increased from 50 to 60. The same degree of increment can, indeed, be observed in Scenario 2 and Scenario 3 of each experiment, in which participants were informed about the values of the α's. When $(\alpha_1, \alpha_2) = (0.5, 0.5)$, the average bid in Scenario 3 is 60.83, whereas with $(\alpha_1, \alpha_2) = (0.2, 0.8)$ the average bid in Scenario 3 increases to 67.08.

We next investigate whether it is proper to apply Assumption 1 as customers' prior during decision making. If customers indeed form the same prior as in Assumption 1, then their bids in Scenario 1 (in which no information is provided and customers have to rely on their own priors to bid) should be similar to Scenario 3 if $(\alpha_1, \alpha_2) = (0.5, 0.5)$. We are, then, interested in testing the following hypothesis:

HYPOTHESIS 1 (**Impact of Full Information**). *Bidding amount should be the same in Scenario 1 and Scenario 3 when $(\alpha_1, \alpha_2) = (0.5, 0.5)$.*

Comparing bids made in Scenario 1 and 3, we find that the p-value is surprisingly close to 1 (see Table 2). Thus, Hypothesis 1 is supported by our experimental results, indicating that the participants likely use the same prior in both scenarios. Thus, the customers own prior mostly likely coincides with the prior in Assumption 1.

We further test if information given in Scenario 3 has been processed during the participants decision-making; that is, whether the similarity of bids in Scenario 1 and Scenario 3 is the effect of poor information utilization.

HYPOTHESIS 2 (**Impact of Full Information**). *Bidding amount should be the same in round 1 and round 3 when* $(\alpha_1, \alpha_2) = (0.2, 0.8)$.

Although the null hypothesis cannot be confidently rejected, the p-value is rather low compared to the earlier scenarios. For example, when $v_1 = 100, v_2 = 130$, the p-value is as low as 0.1379 (that p-value is a little bit higher under $v_1 = 80, v_2 = 90$ could partially be due to the fact that the optimal bids are both \$60 whether (α_1, α_2) are $(0.5, 0.5)$ or $(0.2, 0.8)$). Also, the contrast in average bid is much higher in these scenarios than the earlier two. If we accept that some human subjects may not have properly handled the added information, while other did, then it could reasonably explain why the $(\alpha_1, \alpha_2) = (0.2, 0.8)$ scenario shows less than expected distinction between the rounds.

In addition to exploring customers' subjective priors, the experiment results given in Table 2 offer other interesting insights. For example, one may observe that customers tend to overbid when their valuations are high. Also, bids fluctuate most under half-information compared to no- or full-information. Due to the limited scope of the paper, we leave these interesting behaviour problems to future research.

7.2. Opaque Product Design

The underlying products of the opaque offerings are similar in general. For example, they can be same-star-level hotels in a common region, or flights with the same departing/landing dates. However, in terms of mass consumer valuation, these two cases can be quite different. For hotels, it is natural that some customers may prefer brand A to brand B while there others hold an opposite taste. There is usually no common preference for this kind of products/services. Yet for flights, prime time flights are normally considered better than red-eye flights, both of which may appear in the opaque product realization. Thus, customers hold common preferences upon this type of product/service.

We approximate the above two scenarios by horizontal and vertical differentiation between the two products. In case where no common consensus could be reached upon the superiority of a product, we use horizontal differentiation and assume that $v_{1t} + v_{2t} = \bar{v}_t$ for some constant $\bar{v}_t > 0\}$. When customers commonly prefer one product to the other, we consider vertical differentiation and assume that $v_{1t} - v_{2t} = \bar{v}_t$, for some constant $\bar{v}_t$. It can be shown that these two scenarios have very different implications for the intermediary firm.

PROPOSITION 4. **(Underlying Products: horizontal vs. vertical differentiation)** *(i). If the products are vertically differentiated and $v_{it} > v_{jt}$, then seller i will set a posted price such that all customers purchase from its direct channel. The intermediary NYOP firm will collect zero rents.*

(ii). If the products are horizontally differentiated, then there exists $p_t^0 > 0$ such that (a) when the minimum posted price, p, satisfies $p_t > p_t^0$, all customers will be covered, and the intermediary NYOP firm earns positive revenue; (b) when $p_t \leq p_0^t$, some customers will left empty-handed, and no purchase will be finalized through NYOP.

This implies that, if possible, the intermediary firm should carefully select the underlying products for the opaque offering. If the product candidates have recognized quality difference (a flight departing at 1 a.m. vs. a flight on the same route that leaves at noon) or one product has higher brand recognition (such that all customers may prefer one particular product, although the star levels are the same), then the intermediary firm may not benefit much from these differences. The main reason is that the seller with "better quality/image" can obtain the valuation difference via proper posted price setting, at which the customers feel that the opaque product is not worth bidding for. On the other hand, if the products are horizontally differentiated (e.g., hotels with the same star ranking but different brand-name or small vicinity difference), the valuation for the underlying product will be much more diversified. The sellers would then need the intermediary firm to shoulder part of this uncertainty such that their direct channels can focus on their own target customers. Overall, the intermediary benefits more from horizontally differentiated products than from vertically differentiated ones. This insight conforms with Priceline's 2011 Annual Report to Stockholders (pp.55) that NYOP sales increases in hotel rooms and rental cars, while declining in air tickets.

As the number of sellers increases, there is more leverage for the intermediary firm to determine its supplier base and refine its opaque product design. The firm may also choose whether to inform the customers about the underlying products that comprise the opaque product they are bidding for. There are many interesting research/practical questions with regard to this line of decision making.

7.3. Contracting

Throughout the paper, we consider the intermediary firm as a revenue maximizer, that it always pairs a bid with the seller that charges a lower reservation price. As a result, §6.2 shows that the sellers' channel strategy can take many forms, including that neither will sell through the NYOP intermediary. Therefore, it is worthwhile for the intermediary firm to reconsider the terms and mechanisms in selecting the opaque product provider. In essence, the intermediary should weight the short-run profit earning versus the long-run participation rate of the sellers. These may involve revenue sharing contract between the NYOP firm and the sellers, and selecting opaque provider on a randomized basis rather than a reservation-price basis, etc.

7.4. Information Availability

A key distinction between opaque product and regular product is item-information revelation. It is, then, to the interest of the intermediary firm to explore to what degree should information be offered to its customers. For example, customers purchasing from Hotwire.com can see the other customers' review, amenities, and potential brand names of the blinded hotel. Yet at Priceline.com, fairly little additional information is available for the customers ahead of time. However, for close yet unsuccessful bids, Priceline may give a one-time "counter-offer," hinting the upper bound of the reservation price. Given that the two websites use different pricing mechanism (seller-driven vs. buyer-driven), it is interesting to consider the level and timing of information-revelation under the two settings.

We have also assumed that customers cannot observe real-time inventory levels at the sellers. One may also consider whether it is worthwhile for the sellers to signal their own inventories, an appearing practice in current industry in creating the sense of scarcity, and possibly reducing strategic behaviours as well.

7.5. Social Media

Another promising direction is to consider consumer behaviour under the influence of social media. Today's Internet based applications allow customers to review, discuss, and exchange information beyond traditional online shopping. Specifically for the NYOP channel itself, customers have been sharing successful bidding outcomes on several websites (betterbidding.com, hoteldealsrevealed.com, biddingfortravel.yuku.com, facebook.com, etc.). This apparently can trigger another line of study regarding strategic customer behaviour, which includes social learning, bidding postponement, repeated bidding, etc. Whether the NYOP intermediary should publicize all or part of the recent bids, or analyze the historical bidding data and take any counter action, remains unexplored research questions.

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Appendix

Proof of Proposition 1

For a customer who can afford neither product at listed price ($v_i < p_i$ for $i = 1, 2$), NYOP is the only channel in which she might get some product, so she will always attend NYOP. Her expected payoff is $V_{DC}^B(\mathbf{v}) = \max_b G(b)(\alpha_i v_j + \alpha_i v_j - b)$ The FOC of the inner function is $g(b)(\alpha_1 v_1 + \alpha_2 v_2 - b) - G(b)$. Under DRHR assumption, there exists a unique $b^* \in [0, p]$ that satisfies

$$\alpha_1 v_1 + \alpha_2 v_2 = b^* + G(b^*)/g(b^*). \tag{A1}$$

Also, as $G(b)/g(b)$ increases in b, the optimal bid increases with the expected valuation of the opaque product $\alpha_1 v_1 + \alpha_2 v_2$.

For a customer with an external choice ($v_1 \geq p_1$ and/or $v_2 \geq p_2$), the optimal bid b^* maximizes $G(b)(\alpha_1 v_1 + \alpha_2 v_2 - b) + \bar{G}(b) \max_{k=0,1,2,} V_{DC}^k(\mathbf{v})$. Without loss of generality, assume that $v_1 - p_1 \geq v_2 - p_2$, and denote $|v_i - v_j + p_j - p_i| = v_1 - v_2 + p_2 - p_1 > 0$ the *degree of differentiation*. Followed by a similar analysis as above, it is the unique value that satisfies

$$\alpha_2(v_2 - v_1) + p_1 = b^* + G(b^*)/g(b^*). \tag{A2}$$

Due to DRHR assumption, $b^* \geq 0$ if and only if $\alpha_2(v_2 - v_1) + p_1 \geq 0$. The bid increases with $v_2 - v_1$, hence decreasing in $v_1 - v_2 + p_2 - p_1$, the degree of differentiation.

The customer will choose to bid b^* at the NYOP channel first, if and only if it yields higher expected payoff than buying from direct channel 1, i.e., $V_{DC}^B(\mathbf{v}) \geq V_{DC}^1(\mathbf{v})$, which requires that $G(b^*)(\alpha_1 v_1 + \alpha_2 v_2 - b^*) + \bar{G}(b^*)(v_1 - p_1) > v_1 - p_1$, or equivalently,

$$b^* < (\alpha_1 v_1 + \alpha_2 v_2) - (v_1 - p_1) = \alpha_2(v_2 - v_1) + p_1. \tag{A3}$$

This apparently holds when $\alpha_2(v_2 - v_1) + p_1 \geq 0$. Therefore, the customer will NYOP first if and only if $\alpha_2(v_2 - v_1) + p_1 > 0$, which is equivalent to requiring that the degree of differentiation $v_1 - v_2 + p_2 - p_1 \leq p_1/\alpha_2 + p_2 - p_1 = \dfrac{\alpha_1 p_1 + \alpha_2 p_2}{\alpha_2}$. □

Proof of Theorem 1

Suppose only seller 1 is in stock, the product at the intermediary firm ceases to be an opaque product and becomes a regular one. It is obviously beneficial for the customers to NYOP in the first place, and if rejected, reconsider purchasing from the direct channel. Therefore, the customer first has to determine her bid, b, that maximizes her expected payoff:

$$G(b)(v_j - b) + \bar{G}(b) \max\{v_1 - p_1, 0\} = \begin{cases} G(b)(p - b) + (v_1 - p_1), & \text{if } v_1 \geq p_1 \\ G(b)(v_1 - b), & \text{if } v_1 < p_1. \end{cases}$$

The FOC is given by

$$g(b)\left[\min\{v_1,p_1\}-b\right]-G(b)=g(b)\left[\min\{v_1,p_1\}-b-\frac{G(b)}{g(b)}\right].$$

As $G(b)/g(b)$ increases in b, there exists a unique $b^*\in[0,\min\{v_1,p_1\}]$ such that $\min\{v_1,p_1\}-b^*-\frac{G(b^*)}{g(b^*)}=0$. It is easy to verify that the extreme solutions ($b=0$ and $b=\min\{v,p\}$) are not optimal; therefore, the expected payoff is maximized at FOC=0, i.e, when $b=b^*$. This raises the following lemma:

LEMMA A1. *When only seller 1 is in stock with direct channel price p_1, a customer with valuation v_1 will place a bid b^* with the NYOP channel in the first place, which satisfies*

$$b^*+\frac{G(b^*)}{g(b^*)}=\min\{v_1,p_1\} \tag{A4}$$

By (A4), the optimal bid satisfies $b^*+\frac{G(b^*)}{g(b^*)}=\min\{v_1,p_1\}\le p_1$. Consider r_0 which satisfies

$$r_0+\frac{G(r_0)}{g(r_0)}=p_1. \tag{A5}$$

Then, obviously $b^*<r_0$, i.e., r_0 is beyond the bid of any customer. Therefore if $r\ge r_0$, a customer will be always rejected by the NYOP channel and if $v_1\ge p_1$, she will obtain the product through the direct channel. On the other hand, if $r<r_0$, a customers with $v_1\ge r+\frac{G(r)}{g(r)}$ will bid over r and be accepted by the NYOP intermediary. If one is rejected for bidding below r, i.e., $v_1<r+\frac{G(r)}{g(r)}$, she cannot afford the posted price neither since $v_1<r+\frac{G(r)}{g(r)}<r_0+\frac{G(r_0)}{g(r_0)}=p$. This proves the following lemma:

LEMMA A2. *At period t, suppose r_0 is defined by* (A5) *with $p_1=p_{1t}$. The sales will only be realized through the direct channel if $r_t\ge r_0$, or the NYOP channel otherwise.*

Now consider the seller's optimal decision. With posted price p and reservation price r for a particular period, seller 1's spot revenue π_1 is given by

$$\pi_1(p,r)=\begin{cases}\bar{F}_1(p)p, & \text{if } r>r_0\\ \bar{F}_1\left(r+\frac{G(r)}{g(r)}\right)r & \text{if } r\le r_0,\end{cases}$$

where r_0 is defined in (A5).

At $t=1$, seller 1 will seek a pair (p,r) that maximizes $\pi_1(p,r)$. Denote p^* the optimal posted price if the seller determines to use direct channel, i.e., $r\ge r_0$. If the seller chooses to use the NYOP channel ($r<r_0$) only, the objective becomes

$$\max_{r<r_0}\bar{F}_1\left(r+\frac{G(r)}{g(r)}\right)r=\max_{r<r_0}\frac{r}{r+G(r)/g(r)}\bar{F}_1\left(r+\frac{G(r)}{g(r)}\right)\left(r+\frac{G(r)}{g(r)}\right)$$

$$
\begin{aligned}
&\leq \max_{r<r_0} \frac{r}{r+G(r)/g(r)} F_1(p^*)p^* \\
&< F_1(p^*)p^*.
\end{aligned}
$$

Therefore, in the last time epoch it is always optimal not to let customers purchase from the NYOP channel.

At $t>1$, denote by $\Pi_1(x,t)$ the optimal expected profit that seller 1 will receive if inventory level is x. Then,

$$
\Pi_1(x,t) = \max_{p\geq r\geq 0} \begin{cases} \bar{F}_1(p)\,[\,p+\Pi_1(x-1,t-1)\,]+F_1(p)\Pi_1(x,t-1), & \text{if } r>r_0 \\ \bar{F}_1\left(r+\dfrac{G(r)}{g(r)}\right)[\,r+\Pi_1(x-1,t-1)\,]+F_1\left(r+\dfrac{G(r)}{g(r)}\right)\Pi_1(x,t-1) & \text{if } r\leq r_0, \end{cases}
$$

Let r^* be the optimal reservation price if the seller decides to go with NYOP selling, i.e.,

$$
r^* = \arg\max_r \bar{F}_1\left(r+\frac{G(r)}{g(r)}\right)[\,r+\Pi_1(x-1,t-1)\,]+F_1\left(r+\frac{G(r)}{g(r)}\right)\Pi_1(x,t-1).
$$

Consider $p^* = r^* + \dfrac{G(r^*)}{g(r^*)} - \epsilon < r^* + \dfrac{G(r^*)}{g(r^*)}$, where $\epsilon \to 0$. The following verifies that selling through direct channel with posted price $p_t = p^*$ yields a higher expected profit then that can be achieved via NYOP selling only:

$$
\begin{aligned}
&\bar{F}_1(p^*)\,[\,p^*+\Pi_1(x-1,t-1)\,]+F_1(p^*)\Pi_1(x,t-1) \\
=\;&\bar{F}_1\left(r^*+\frac{G(r^*)}{g(r^*)}\right)[r^*+\Pi_1(x-1,t-1)]+F_1\left(r^*+\frac{G(r^*)}{g(r^*)}\right)\Pi_1(x,t-1)+\bar{F}_1\left(r^*+\frac{G(r^*)}{g(r^*)}\right)\frac{G(r^*)}{g(r^*)} \\
&+\left[\bar{F}_1\left(r^*+\frac{G(r^*)}{g(r^*)}-\epsilon\right)-\bar{F}_1\left(r^*+\frac{G(r^*)}{g(r^*)}\right)\right]\left[r^*+\frac{G(r^*)}{g(r^*)}+\Pi_1(x-1,t-1)-\Pi_1(x,t-1)\right] \\
&-\epsilon\bar{F}_1\left(r^*+\frac{G(r^*)}{g(r^*)}-\epsilon\right).
\end{aligned}
$$

When ϵ is small enough, we have

$$
\begin{aligned}
&\left[\bar{F}_1\left(r^*+\frac{G(r^*)}{g(r^*)}-\epsilon\right)-\bar{F}_1\left(r^*+\frac{G(r^*)}{g(r^*)}\right)\right]\left[r^*+\frac{G(r^*)}{g(r^*)}+\Pi_1(x-1,t-1)-\Pi_1(x,t-1)\right] \\
&-\epsilon\bar{F}_1\left(r^*+\frac{G(r^*)}{g(r^*)}-\epsilon\right)\to 0.
\end{aligned}
$$

Hence,

$$
\begin{aligned}
&\bar{F}_1(p^*)\,[\,p^*+\Pi_1(x-1,t-1)\,]+F_1(p^*)\Pi_1(x,t-1) \\
&\geq \bar{F}_1\left(r^*+\frac{G(r^*)}{g(r^*)}\right)[r^*+\Pi_1(x-1,t-1)]+F_1\left(r^*+\frac{G(r^*)}{g(r^*)}\right)\Pi_1(x,t-1).
\end{aligned}
$$

Therefore, it is optimal for seller 1 to use direct channel only.

We characterize the optimal price and expected profit as follows:

For $t=1$, $p^*_{11}=\arg\max_p \bar{F}_1(p)p=\arg\max_p(1-p)p=1/2$, $\Pi_1(x,1)=\bar{F}_1(p^*_{11})p^*_{11}=1/4$.

For $t>1$,

$$\begin{aligned}
p^*_{1t}(x) &= \arg\max_p \bar{F}_1(p)\,[p+\Pi_1(x-1,t-1)]+F_1(p)\Pi_1(x,t-1)\\
&= \arg\max_p [1-p]\,[p+\Pi_1(x-1,t-1)]+p\Pi_1(x,t-1)\\
&= \arg\max_p -p^2+p[\,1+\Pi_1(x,t-1)-\Pi(x-1,t-1)\,]+\Pi_1(x-1,t-1)\\
&= \frac{1+\Pi_1(x,t-1)-\Pi_1(x-1,t-1)}{2}
\end{aligned}$$

and

$$\begin{aligned}
\Pi_1^*(x,t) &= \bar{F}_1(p^*_{1t})\,[p^*_{1t}+\Pi_1(x-1,t-1)]+F_1(p^*_{1t})\Pi_1(x,t-1)\\
&= \Pi_1(x-1,t-1)+\left(\frac{1+\Pi_1(x,t-1)-\Pi_1(x-1,t-1)}{2}\right)^2
\end{aligned}$$

□

Proof of Theorem 2: From §5.2, we only need to show the existence of equilibrium on (p_{1t},p_{2t}) when seller 1 choose no to sell through NYOP (i.e., $r^*_{1t}=1$, $\Pi^*_{1t}=\Pi^D_{1t}$) in period t. In particular, when $r_{1t}=r_{2t}=1$,

$$\begin{aligned}
\Pi_1^D(\mathbf{x},t) &= [\,p_{1t}+\Pi_1(x_1-1,x_2,t-1)\,]\,\Omega_1(\mathbf{r}_t,\mathbf{p}_t)+\Pi_1(x_1,x_2-1,t-1)\Omega_2(\mathbf{r}_t,\mathbf{p}_t)+\Pi_1(x_1,x_2,t-1)\Omega_0(\mathbf{r}_t,\mathbf{p}_t)\\
\Pi_2(\mathbf{x},t) &= \Pi_2(x_1-1,x_2,t-1)\Omega_1(\mathbf{r}_t,\mathbf{p}_t)+[\,p_{2t}+\Pi_2(x_1,x_2-1,t-1)\,]\,\Omega_2(\mathbf{r}_t,\mathbf{p}_t)+\Pi_2(x_1,x_2,t-1)\Omega_0(\mathbf{r}_t,\mathbf{p}_t)
\end{aligned}$$

We need to prove that Π_1^D and Π_2 are unimodals in p_{1t} and p_{2t} respectively. Note that for $i=1,2$,

$$\frac{\partial\Omega_1(\mathbf{r}_t,\mathbf{p}_t)}{\partial p_i}+\frac{\partial\Omega_2(\mathbf{r}_t,\mathbf{p}_t)}{\partial p_i}+\frac{\partial\Omega_0(\mathbf{r}_t,\mathbf{p}_t)}{\partial p_i}=0.$$

Thus,

$$\begin{aligned}
\frac{\partial\Pi_1^D(\mathbf{x},t)}{\partial p_{1t}} &= \Omega_1(\mathbf{r}_t,\mathbf{p}_t)\\
&\quad -\left[\widetilde{\Pi}_1(x_1,x_2,t-1)-p_{1t}\right]\frac{\partial\Omega_1(\mathbf{r}_t,\mathbf{p}_t)}{\partial p_{1t}}-[\Pi_1(x_1,x_2-1,t-1)-\Pi_1(x_1,x_2,t-1)]\,\frac{\partial\Omega_0(\mathbf{r}_t,\mathbf{p}_t)}{\partial p_{1t}}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial\Pi_2(\mathbf{x},t)}{\partial p_{2t}} &= \Omega_2(\mathbf{r}_t,\mathbf{p}_t)\\
&\quad -\left[\widetilde{\Pi}_2(x_1,x_2,t-1)-p_{2t}\right]\frac{\partial\Omega_2(\mathbf{r}_t,\mathbf{p}_t)}{\partial p_{2t}}-[\Pi_2(x_1-1,x_2,t-1)-\Pi_2(x_1,x_2,t-1)]\,\frac{\partial\Omega_0(\mathbf{r}_t,\mathbf{p}_t)}{\partial p_{2t}}
\end{aligned}$$

Under Assumption 1, (1) becomes

$$\mathbf{H}_{SDC1}(\mathbf{v}) = \begin{cases} (0,0) & \text{if } \max\{v_1 - p_1, v_2 - p_2\} < 0 \text{ and } 2r > v_1; \\ (1,p_1) & \text{if } v_1 - p_1 \geq \max\{v_2 - p_2, 0\} \text{ and } p_1 - 2r < 0; \\ (2,p_2) & \text{if } v_2 - p_2 \geq \max\{v_1 - p_1, 0\} \text{ and } p_2 - 2r < v_2 - v_1; \\ (O,b^*) & \text{otherwise.} \end{cases} \tag{A6}$$

For $r = 1$, it can be verified that if $p_i \leq p_j$,

$$\Omega_i(\mathbf{r},\mathbf{p}) = 1 - \frac{2p_i + (1-p_j)^2}{2}, \ \Omega_j(\mathbf{r},\mathbf{p}) = \frac{(2p_i + 1 - p_j)(1-p_j)}{2}, \ \Omega_0(\mathbf{r},\mathbf{p}) = p_1p_2, \ \Omega_O(\mathbf{r},\mathbf{p}) = 0$$

and

$$\begin{aligned} &\frac{\partial\Omega_i(\mathbf{r}_t,\mathbf{p}_t)}{\partial p_{it}} = -1, \quad \frac{\partial\Omega_j(\mathbf{r}_t,\mathbf{p}_t)}{\partial p_{it}} = 1 - p_j, \quad \frac{\partial\Omega_0(\mathbf{r}_t,\mathbf{p}_t)}{\partial p_{it}} = p_j; \\ &\frac{\partial\Omega_i(\mathbf{r}_t,\mathbf{p}_t)}{\partial p_{jt}} = 1 - p_j, \quad \frac{\partial\Omega_j(\mathbf{r}_t,\mathbf{p}_t)}{\partial p_{jt}} = p_j - 1, \quad \frac{\partial\Omega_0(\mathbf{r}_t,\mathbf{p}_t)}{\partial p_{jt}} = p_i; \\ &\frac{\partial^2\Omega_i(\mathbf{r}_t,\mathbf{p}_t)}{\partial p_{it}^2} = 0, \quad \frac{\partial^2\Omega_j(\mathbf{r}_t,\mathbf{p}_t)}{\partial p_{it}^2} = 0, \quad \frac{\partial^2\Omega_0(\mathbf{r}_t,\mathbf{p}_t)}{\partial p_{it}^2} = 0; \\ &\frac{\partial^2\Omega_i(\mathbf{r}_t,\mathbf{p}_t)}{\partial p_{jt}^2} = -1, \quad \frac{\partial^2\Omega_j(\mathbf{r}_t,\mathbf{p}_t)}{\partial p_{jt}^2} = 1, \quad \frac{\partial^2\Omega_0(\mathbf{r}_t,\mathbf{p}_t)}{\partial p_{jt}^2} = 0 \end{aligned}$$

Without loss of generality, assume that $i = 1$ and $j = 2$. Then

$$\begin{aligned} \frac{\partial\Pi_1^D(\mathbf{x},t)}{\partial p_{1t}} &= 1 - p_{1t} - \frac{(1-p_{2t})^2}{2} \\ &\quad + \left[\widetilde{\Pi_1}(x_1,x_2,t-1) - p_{1t}\right] - \left[\Pi_1(x_1,x_2-1,t-1) - \Pi_1(x_1,x_2,t-1)\right]p_{2t} \\ \frac{\partial\Pi_2(\mathbf{x},t)}{\partial p_{2t}} &= \frac{(2p_{1t}+1-p_{2t})(1-p_{2t})}{2} \\ &\quad + \left[\widetilde{\Pi_2}(x_1,x_2,t-1) - p_{2t}\right](1-p_{2t}) - \left[\Pi_2(x_1-1,x_2,t-1) - \Pi_2(x_1,x_2,t-1)\right]p_{1t} \end{aligned}$$

Apparently, Π_i^D is unimodal as $\dfrac{\partial\Pi_1^D(\mathbf{x},t)}{\partial p_{1t}}$ can take the value 0 at at most one p_{1t}. For Π_2, the FOC can be written as

$$\frac{\partial\Pi_2(\mathbf{x},t)}{\partial p_{2t}} = \frac{3}{2}(1-p_{2t})^2 + \left[\widetilde{\Pi}_2(x_1,x_2,t-1) + \frac{p_{1t}}{2} - 1\right](1-p_{2t}) - \left[\Pi_2(x_1-1,x_2,t-1) - \Pi_2(x_1,x_2,t-1)\right]p_{1t}$$

Note that $\Pi_2(x_1-1,x_2,t-1) - \Pi_2(x_1,x_2,t-1) \geq 0$, therefore, one root (if any) of the FOC will satisfy $p_{2t} > 1$. Thus, Π_2 as a function of p_{2t} is unimodal on $[0,1]$. These prove the existence of pure strategy NE. □

Proof of Proposition 2: For stage 2 at time t, the seller with lower reservation price r_i will be chosen by the NYOP intermediary as the opaque product provider. Consider seller 1, suppose his rival seller 2 proposes a reservation price $r_{2t} \geq \widetilde{\Pi_1}(\mathbf{x},t-1)$. Then, seller 1 can practically take three sets of actions:

1. *give up* the opaque provider-ship by setting $r_{1t}=1$. In this case, seller 1's expected revenue can be expressed by

$$\begin{aligned}\Pi_1^{>r}(\mathbf{p}|\mathbf{x},t) &= [\,p_1+\Pi_1(x_1-1,x_2,t-1)\,]\,\Omega_1(\mathbf{r}_t,\mathbf{p}_t)+\Pi_1(x_1,x_2-1,t-1)\Omega_O(\mathbf{r}_t,\mathbf{p}_t)\\ &\quad+\Pi_1(x_1,x_2-1,t-1)\Omega_2(\mathbf{r}_t,\mathbf{p}_t)+\Pi_1(x_1,x_2,t-1)\Omega_0(\mathbf{r}_t,\mathbf{p}_t);\end{aligned}$$

2. *match* the reservation price by setting $r_{1t}=r_{2t}$. In this case, each seller is the opaque provider with equal probability.

$$\begin{aligned}&\Pi_1^{=r}(\mathbf{p}|\mathbf{x},t)=[\,p_1+\Pi_1(x_1-1,x_2,t-1)\,]\,\Omega_1(\mathbf{r}_t,\mathbf{p}_t)\\ &+\left\{\frac{1}{2}\,[\,r+\Pi_1(x_1-1,x_2,t-1)\,]+\frac{1}{2}\Pi_1(x_1,x_2-1,t-1)\right\}\Omega_O(\mathbf{r}_t,\mathbf{p}_t)\\ &+\Pi_1(x_1,x_2-1,t-1)\Omega_2(\mathbf{r}_t,\mathbf{p}_t)+\Pi_1(x_1,x_2,t-1)\Omega_0(\mathbf{r}_t,\mathbf{p}_t);\end{aligned}$$

3. *become* the opaque provider by agreeing to a lower reservation price $r_{1t}=r_{2t}-\epsilon$. As $\epsilon\to 0$, the expected revenue is given by

$$\begin{aligned}\Pi_1^{<r}(\mathbf{p}|\mathbf{x},t) &= [\,p_1+\Pi_1(x_1-1,x_2,t-1)\,]\,\Omega_1(\mathbf{r}_t,\mathbf{p}_t)+[\,r+\Pi_1(x_1-1,x_2,t-1)\,]\,\Omega_O(\mathbf{r}_t,\mathbf{p}_t)\\ &\quad+\Pi_1(x_1,x_2-1,t-1)\Omega_2(\mathbf{r}_t,\mathbf{p}_t)+\Pi_1(x_1,x_2,t-1)\Omega_0(\mathbf{r}_t,\mathbf{p}_t).\end{aligned}$$

The first, third and fourth items for each revenue function describe the expected income if the arriving customer at time t will buy from direct channel 1, 2, or empty handed, respectively. These three components stay the same across each strategy. The second item represents the expected income if the next arrival is an NYOP buyer. It is the only part that varies according to the opaque strategy of seller 1. Comparing the three strategies, it is not hard to verify that it is optimal for seller i to *become* opaque provider whenever $r_{2t}>\widetilde{\Pi_1}(\mathbf{x},t-1)$, *match* when $r_{2t}=\widetilde{\Pi_1}(\mathbf{x},t-1)$, and *give up* when $r_{2t}<\widetilde{\Pi_1}(\mathbf{x},t-1)$.

Without loss of generality, assume that $\widetilde{\Pi_1}(\mathbf{x},t-1)\le\widetilde{\Pi_2}(\mathbf{x},t-1)$. Then, seller 2 will compete with seller 1 for the opaque provider-ship until $r_{1t}=r_{2t}=\widetilde{\Pi_2}(\mathbf{x},t-1)$. And seller 1 can gain the full provider-ship by lowering down his reservation price to $\widetilde{\Pi_2}(\mathbf{x},t-1)-\epsilon$ where $\epsilon\to 0$. Therefore, the minimum reservation price is $r^*(\mathbf{x},t)=\widetilde{\Pi_2}(\mathbf{x},t-1)$. Seller 1 will be the unique opaque provider unless $\widetilde{\Pi_1}(\mathbf{x},t-1)=\widetilde{\Pi_2}(\mathbf{x},t-1)$, in which case both sellers are the opaque provider with equal probability. □

Proof of Proposition 3: At $t=1$, the marginal value of inventory $\widetilde{\Pi_i}(\mathbf{x},t-1)$ is zero for either seller. By Proposition 2, the reservation price is $r^*=\max_i\widetilde{\Pi_i}(\mathbf{x},t-1)=0$ □

Proof of Theorem 3: We aim to prove that Π_i is unimodal in p_{it} for $i=1,2$. Without loss of generality, assume that $r_t = \widetilde{\Pi_2}(\mathbf{x}, t-1) < \widetilde{\Pi_1}(\mathbf{x}, t-1)$, i.e., seller 2 is the opaque-product provider. Then,

$$\begin{aligned}\Pi_1(\mathbf{x},t) &= [p_{1t}+\Pi_1(x_1-1,x_2,t-1)]\,\Omega_1(\mathbf{r}_t,\mathbf{p}_t)+\Pi_1(x_1,x_2-1,t-1)\Omega_O(\mathbf{r}_t,\mathbf{p}_t)\\ &\quad +\Pi_1(x_1,x_2-1,t-1)\Omega_2(\mathbf{r}_t,\mathbf{p}_t)+\Pi_1(x_1,x_2,t-1)\Omega_0(\mathbf{r}_t,\mathbf{p}_t) \qquad \text{(A7a)}\\ \Pi_2(\mathbf{x},t) &= \Pi_2(x_1-1,x_2,t-1)\Omega_1(\mathbf{r}_t,\mathbf{p}_t)+\left[\widetilde{\Pi_2}(\mathbf{x},t-1)+\Pi_2(x_1,x_2-1,t-1)\right]\Omega_O(\mathbf{r}_t,\mathbf{p}_t)\\ &\quad +[p_{2t}+\Pi_2(x_1,x_2-1,t-1)]\,\Omega_2(\mathbf{r}_t,\mathbf{p}_t)+\Pi_2(x_1,x_2,t-1)\Omega_0(\mathbf{r}_t,\mathbf{p}_t) \qquad \text{(A7b)}\end{aligned}$$

Note that

$$-\frac{\partial\Omega_O(\mathbf{r}_t,\mathbf{p}_t)}{\partial p_i} = \frac{\partial\Omega_1(\mathbf{r}_t,\mathbf{p}_t)}{\partial p_i}+\frac{\partial\Omega_2(\mathbf{r}_t,\mathbf{p}_t)}{\partial p_i}+\frac{\partial\Omega_0(\mathbf{r}_t,\mathbf{p}_t)}{\partial p_i}$$

for $i=1,2$. Then,

$$\begin{aligned}\frac{\partial\Pi_1}{\partial p_{1t}} &= \Omega_1-\left(\widetilde{\Pi_1}-p_{1t}\right)\frac{\partial\Omega_1}{\partial p_{1t}}-\Delta\Pi_1\frac{\partial\Omega_0}{\partial p_{1t}}\\ \frac{\partial^2\Pi_1}{\partial p_{1t}^2} &= 2\frac{\partial\Omega_1}{\partial p_{1t}}-\left[\widetilde{\Pi_1}-p_{1t}\right]\frac{\partial^2\Omega_1}{\partial p_{1t}^2}-\Delta\Pi_1\frac{\partial^2\Omega_0}{\partial p_{1t}^2}.\\ \frac{\partial\Pi_2}{\partial p_{2t}} &= \Omega_2-\left(\widetilde{\Pi_2}-p_{2t}\right)\frac{\partial\Omega_2}{\partial p_{2t}}-\Delta\Pi_2\frac{\partial\Omega_0}{\partial p_{2t}}\\ \frac{\partial^2\Pi_2}{\partial p_{2t}^2} &= 2\frac{\partial\Omega_2}{\partial p_{2t}}-\left[\widetilde{\Pi_2}-p_{2t}\right]\frac{\partial^2\Omega_2}{\partial p_{2t}^2}-\Delta\Pi_2\frac{\partial^2\Omega_0}{\partial p_{2t}^2}.\end{aligned}$$

For the ease of presentation, we omit all the function variables, and use $\Delta\Pi_1$ and $\Delta\Pi_2$ to denote $\Pi_1(x_1,x_2-1,t-1)-\Pi_1(x_1,x_2,t-1)$ and $\Pi_2(x_1-1,x_2,t-1)-\Pi_2(x_1,x_2,t-1)$ respectively. The same notations apply to the rest of the proof.

We first prove that Π_1 is unimodal in p_{1t}. Under Assumption 1, $\mathbf{H}_{DC}(\mathbf{v})$ can be rewritten as follows:

$$\mathbf{H}_{DC}(\mathbf{v}) = \begin{cases}(0,0) & \text{if } \max\{v_1-p_1, v_2-p_2\}<0 \text{ and } v_1+v_2\le 4r;\\ (1,p_1) & \text{if } v_1-p_1\ge\max\{v_2-p_2,0\} \text{ and } v_1-v_2\ge 2p_1-4r;\\ (2,p_2) & \text{if } v_2-p_2\ge\max\{v_1-p_1,0\} \text{ and } v_2-v_1\ge 2p_2-4r;\\ (O,b^*) & \text{otherwise.}\end{cases} \qquad \text{(A8)}$$

To avoid trivial cases, assume that $p_{2t}\le 4r_t\le 1$ (the proof could be done in a similar fashion otherwise). Depending on the value of p_{1t}, its first order effect on Π_1 can take the following forms:

I. When $p_{1t}\le p_{2t}<4r_t-p_{2t}$, there are $\Omega_1 = 1-\dfrac{2p_{1t}+(1-p_{2t})^2}{2}$ and $\Omega_0 = p_{1t}p_{2t}$. Hence,

$$\frac{\partial\Pi_1(\mathbf{x},t)}{\partial p_{1t}} = 1-2p_{1t}-\frac{(1-p_{2t})^2}{2}+\widetilde{\Pi_1}-\Delta\Pi_1 p_{2t}$$

II. When $p_{2t} \le p_{1t} < 4r_t - p_{2t}$, there are $\Omega_1 = \dfrac{(2p_{2t}+1-p_{1t})(1-p_{1t})}{2}$ and $\Omega_0 = p_{1t}p_{2t}$. Thus,

$$\frac{\partial \Pi_1(\mathbf{x},t)}{\partial p_{1t}} = \frac{3}{2}(1-p_{1t})^2 + (2p_{2t}+\widetilde{\Pi}_1 - 1)(1-p_{1t}) - p_{2t}(1+\Delta\Pi_1 - \widetilde{\Pi}_1)$$

III. When $\max\{4r_t - p_{2t}, p_{2t}\} \le p_{1t} < 4r_t$, there are $\Omega_1 = \dfrac{3p_{1t}^2 - (4+8r_t)p_{1t} + 1 + 8r_t}{2}$ and $\Omega_0 = \dfrac{-p_{1t}^2 - p_{2t}^2 - 16r_t^2 + 8r_t(p_{1t}+p_{2t})}{2}$. Hence,

$$\frac{\partial \Pi_1(\mathbf{x},t)}{\partial p_{1t}} = \frac{9}{2}p_{1t}^2 - (4+8r_t+3\widetilde{\Pi}_1 - \Delta\Pi_1)p_{1t} + \frac{1}{2} + 4r_t + (4r_t+2)\widetilde{\Pi}_1 - 4r_t\Delta\Pi_1$$

IV. When $4r_t \le p_{1t} < \dfrac{1+4r_t}{2}$, there are $\Omega_1 = \dfrac{(1+4r-2p_{1t})^2}{2}$ and $\Omega_0 = \dfrac{(8r-p_{2t})p_{2t}}{2}$. Thus,

$$\frac{\partial \Pi_1(\mathbf{x},t)}{\partial p_{1t}} = \frac{(1+4r_t-2p_{1t})^2}{2} + 2(\widetilde{\Pi}_1 - p_{1t})(1+4r_t-2p_{1t})$$

V. When $\dfrac{1+4r_t}{2} \le p_{1t} \le 1$, there are $\Omega_1 = 0$ and $\Omega_0 = \dfrac{(8r-p_{2t})p_{2t}}{2}$. Thus, $\dfrac{\partial \Pi_1(\mathbf{x},t)}{\partial p_{1t}} = 0$.

We next prove the unimodal by showing that there exists an $p_{1t}^* \in [0,1]$ such that $\dfrac{\partial \Pi_1(\mathbf{x},t)}{\partial p_{1t}} \ge 0$ if $0 \le p_{1t} \le p_{1t}^*$, and $\dfrac{\partial \Pi_1(\mathbf{x},t)}{\partial p_{1t}} \le 0$ if $p_{1t}^* \le p_{1t} \le 1$.

First note that $\dfrac{\partial \Pi_1(\mathbf{x},t)}{\partial p_{1t}}$ is continuous in Scenario I and II. In Scenario I, it decreases in p_{1t} and

$$\frac{\partial \Pi_1(\mathbf{x},t)}{\partial p_{1t}}|_{p_{1t}=0} = \frac{1}{2} - \frac{p_{2t}^2}{2} + (1-\Delta\Pi_1)p_{2t} + \widetilde{\Pi}_1 > 0.$$

In Scenario II, $\dfrac{\partial \Pi_1(\mathbf{x},t)}{\partial p_{1t}}$ is convex. Since $-p_{2t}(1+\Delta\Pi_1 - \widetilde{\Pi}_1) < 0$, at least one (if any) of the roots of the first-order condition (FOC) should be greater than 1. Also, as

$$\frac{\partial \Pi_1(\mathbf{x},t)}{\partial p_{1t}}|_{p_{1t}=0} = \frac{1}{2} + 4r_t + (4r_t+2)\widetilde{\Pi}_1 - 4r_t\Delta\Pi_1 > 0,$$

there is one $p_{1t} \in [0,1]$ at which the FOC is achieved, denoted as p_{1t}^{II}.

In Scenario III, $\dfrac{\partial \Pi_1(\mathbf{x},t)}{\partial p_{1t}}$ is convex and

$$\begin{aligned}\frac{\partial \Pi_1(\mathbf{x},t)}{\partial p_{1t}}|_{p_{1t}=0} &= \frac{1}{2} + 4r_t + (4r_t+2)\widetilde{\Pi}_1 - 4r_t\Delta\Pi_1 > 0\\ \frac{\partial \Pi_1(\mathbf{x},t)}{\partial p_{1t}}|_{p_{1t}=1} &= (1-4r_t)(1-\widetilde{\Pi}_1 - \Delta\Pi_1) < 0\end{aligned}$$

Thus, there is one $p_{1t} \in [0,1]$ at which FOC is achieved, denoted as p_{1t}^{III}.

In scenario IV, the FOC can be achieved at $p_{1t} = \dfrac{1+4r_t+4\widetilde{\Pi}_1}{6}$ and $p_{1t} = \dfrac{1+4r_t}{2}$, where $\dfrac{1+4r_t+4\widetilde{\Pi}_1}{6} \le \dfrac{1+4r_t}{2}$. Denote $p_{1t}^{IV} = \dfrac{1+4r_t+4\widetilde{\Pi}_1}{6}$. If $p_{1t}^{IV} \le 4r_t$, then $\dfrac{\partial \Pi_1(\mathbf{x},t)}{\partial p_{1t}}$ is constantly negative in Scenario IV. Otherwise, it can be shown that $\frac{\partial \Pi_1(\mathbf{x},t)}{\partial p_{1t}}$ in Scenario III will be 0 at $p_{1t}=4r_t$, i.e., $p_{1t}^{III}=4r_t$.

Thus, p_{1t}^* can only be among p_{1t}^{II}, p_{1t}^{III} and p_{1t}^{IV}. It can be shown that p_{1t}^{II} and p_{1t}^{III} cannot both be feasible for the respective scenarios at the same time. That is, $p_{1t}^{II} \le 4r_t - p_{2t} \le p_{1t}^{III}$ cannot hold true. Otherwise, consider the boundary point where $4r_t = p_{1t}+p_{2t}$. Some algebra leads to that

$$\frac{\partial \Pi_1(\mathbf{x},t)}{\partial p_{1t}}^{II}\Big|_{p_{1t}=4r_t-p_{2t}} - \frac{\partial \Pi_1(\mathbf{x},t)}{\partial p_{1t}}^{III}\Big|_{p_{1t}=4r_t-p_{2t}} = (p_{1t}-\widetilde{\Pi}_1)(1-p_{1t})$$

Due to the definition of $\widetilde{\Pi}_1$, $p_{1t}\ge\widetilde{\Pi}_1$ should be satisfied at both p_{1t}^{II} and p_{1t}^{III}. Therefore, $\frac{\partial \Pi_1(\mathbf{x},t)}{\partial p_{1t}}^{II}\Big|_{p_{1t}=4r_t-p_{2t}} \ge \frac{\partial \Pi_1(\mathbf{x},t)}{\partial p_{1t}}^{III}\Big|_{p_{1t}=4r_t-p_{2t}}$. However, for what we have discussed about $\frac{\partial \Pi_1(\mathbf{x},t)}{\partial p_{1t}}$, there should be $\frac{\partial \Pi_1(\mathbf{x},t)}{\partial p_{1t}}^{III}\Big|_{p_{1t}=4r_t-p_{2t}} \ge \frac{\partial \Pi_1(\mathbf{x},t)}{\partial p_{1t}}^{III}\Big|_{p_{1t}=p_{1t}^{III}} = 0 = \frac{\partial \Pi_1(\mathbf{x},t)}{\partial p_{1t}}^{II}\Big|_{p_{1t}=p_{1t}^{II}} \ge \frac{\partial \Pi_1(\mathbf{x},t)}{\partial p_{1t}}^{II}\Big|_{p_{1t}=4r_t-p_{2t}}$.

Thus far, it can be concluded that if $p_{1t}^{III}\le 4r_t-p_{2t}$, $\frac{\partial \Pi_1(\mathbf{x},t)}{\partial p_{1t}}\le 0$ in Scenario III, IV and V, thus $p_{1t}^*=\max\{p_{1t}^{II}, 4r_t-p_{2t}\}$. If $4r_t-p_{2t}\le p_{1t}^{III}<4r_t$, $\frac{\partial \Pi_1(\mathbf{x},t)}{\partial p_{1t}}\le 0$ in Scenario IV and V, and $\frac{\partial \Pi_1(\mathbf{x},t)}{\partial p_{1t}}\ge 0$ in Scenario II, hence $p_{1t}^*=p_{1t}^{III}$. If $4r_t\le p_{1t}^{III}$, $\frac{\partial \Pi_1(\mathbf{x},t)}{\partial p_{1t}}\ge 0$ in Scenario II, III and IV when $p_{1t}\le p_{1t}^{IV}$, and $\frac{\partial \Pi_1(\mathbf{x},t)}{\partial p_{1t}}\le 0$ when $p_{1t}>p_{1t}^{IV}$; hence $p_{1t}^*=p_{1t}^{IV}$.

The above proves the existence of p_{1t}^* thus $\Pi_1(\mathbf{x},t)$ is unimodal in p_{1t}. That for $\Pi_2(\mathbf{x},t)$ can be done in a similar fashion. Therefore, there exists a pure NE for the posted prices (p_{1t},p_{2t}). □

Proof of Proposition 3: We can show that, at any time t, if $x_1\ge t$ and $x_2\ge t$,

$$\Pi_1(x_1,x_2,t)=\Pi_2(x_1,x_2,t)=\pi_t \tag{A9}$$

for some $\pi_t>0$. Hence $\widetilde{\Pi}_i(\mathbf{x},t)=0$ for any $x_1\ge t$ and $x_2\ge t$, which immediately leads to $r^*=0$.

We show (A9) by induction. First, it is apparent that (A9) holds for $t=0$. Now, suppose (A9) holds for all $t<T$. We then have $\Pi_1(x_1-1,x_2,T-1)=\Pi_1(x_1-1,x_2-1,T-1)=\Pi_2(x_1,x_2-1,T-1)=\Pi_2(x_1-1,x_2-1,T-1)$ for any $x_1\ge T$ and $x_2\ge T$. Thus, $\widetilde{\Pi_i}(x_1,x_2,T-1)=0$ for $i=1,2$ and

$$r^*(x_1,x_2,T)=0, \qquad \forall x_1\ge T, x_2\ge T. \tag{A10}$$

(A10) implies that, when both sellers oversupply, the price of the opaque goods will remain zero until one seller's inventory becomes lower than the potential demand. The expected profit for seller 1 is then

$$\Pi_1(\mathbf{x},T) = [\,p_{1T}+\Pi_1(x_1-1,x_2,T-1)\,]\,\Omega_1(\mathbf{r}_t,\mathbf{p}_t)+\Pi_1(x_1-1,x_2,T-1)\Omega_O(\mathbf{r}_t,\mathbf{p}_t)$$

$$+\Pi_1(x_1, x_2-1, T-1)\Omega_2(\mathbf{r}_t, \mathbf{p}_t) + \Pi_1(x_1, x_2, T-1)\Omega_0(\mathbf{r}_t, \mathbf{p}_t)$$
$$= p_{1T}\Omega_1(\mathbf{r}_t, \mathbf{p}_t) + \pi_{T-1},$$

which does not depend on $\mathbf{x}$ for $x_1 \geq T$ and $x_2 \geq T$. Similarly, $\Pi_2(\mathbf{x}, T) = p_{2T}\Omega_2(\mathbf{r}_t, \mathbf{p}_t) + \pi_{T-1}$. When $r = 0$, $\Omega_i(\mathbf{r}_t, \mathbf{p}_t) = \frac{(1-2p_{it})^2}{2}$ for $i = 1, 2$. It is then straightforward that in equilibrium there should be $p_{1T} = p_{2T}$ and $\Pi_1(\mathbf{x}, T) = \Pi_2(\mathbf{x}, T)$. Thus, (A9) also holds for $t = T$. This completes the proof. □

Proof of Proposition 4:

(i) For vertical differentiation, consider that $\{\mathbf{v}_t : v_{it} - v_{jt} = \bar{v}_t\}$ for some constant $\bar{v}_t > 0$. By $\mathbf{H}_{DC}(\mathbf{v})$ in §4.3, a customer will consistently buy from the *same* type of channel (i.e., from seller i, or seller j, or the NYOP channel) or leave empty handed. (*)

• Consider $t = 1$, the last period sales. Due to Theorem 1, the statement apparently holds when one seller is out of stock. Now, suppose both sellers are in stock. Proposition 3 implies that both sellers has marginal inventory value zero thus $r_1 = 0$ hence no seller earns from the NYOP channel. By (*), sellers' should set posted prices to induce the customer purchase through direct channel ultimately.

We first argue that a customer will not buy directly from seller j. First, there is always a strategy in which seller i can set his posted prices as $p_{i1} = p_{j1} + v_{i1} - v_{j1} - \varepsilon = p_{j1} + \bar{v}_t - \varepsilon$ for some $\varepsilon > 0$ such that the last-period customer will always prefer to buy from seller i at p_{i1}. Second, it is optimal for seller i to set such a posted price in winning the potential customer, since direct channel is the only place that generates income. The competition then drives the period-1 posted prices for i and j to $p_{i1} = \bar{v}_1 - \varepsilon$ and $p_{j1} = 0$ respectively.

Next, to ensure that the customer will prefer buying directly from seller i than obtaining opaque product from NYOP channel, the posted price for i should also satisfy $\alpha_i v_{i1} + \alpha_j v_{j1} \leq v_{i1} - p_{i1}$, hence $p_{i1} \leq \alpha_j \bar{v}_1$. The equilibrium at $t = 1$ is therefore $p_{i1} = \alpha_j \bar{v}_1$ and $p_{j1} = 0$. The expected profit for seller i is $\alpha_j \bar{v}_1$. seller j and the NYOP firm earns zero.

The statement is true for $t = 1$.

• Now consider $t > 1$. Denote $v^r_{it} = r^*_t + \frac{G(r^*_t)}{g(r^*_t)} + \alpha_j \bar{v}_t$ and $v^r_{jt} = r^*_t + \frac{G(r^*_t)}{g(r^*_t)} - \alpha_i \bar{v}_t$. There is $v^r_{it} - v^r_{jt} = \bar{v}_t$. Then, if $v_{it} < p_{it}$ and $v_{jt} < p_{jt}$, customers with $v_{it} \geq v^r_{it}$ or $v_{jt} \geq v^r_{jt}$ will bid above the reservation price r^*_t and purchase through the NYOP channel. Other customers cannot afford either direct channel and will leave empty handed.

If $v_{it} \geq p_{it}$ and $\bar{v}_t \geq p_{it} - p_{jt}$, analysis in §4.3 suggests that the optimal bid satisfies $\min\{v_{it}, p_{it}\} = b^* + \frac{G(b^*)}{g(b^*)} + \alpha_j \bar{v}_t$. Thus those with $\min\{v_{it}, p_{it}\} < v^r_{it}$ will bid below r^*_t and will purchase through direct channel i in the end.

If $v_{jt} \geq p_{jt}$ and $\bar{v}_t \leq p_{it} - p_{jt}$, the optimal bid satisfies $\min\{v_{jt}, p_{jt}\} = b^* + \dfrac{G(b^*)}{g(b^*)} - \alpha_i \bar{v}_t$, and those with $\min\{v_{jt}, p_{jt}\} < v^r_{jt}$ will bid below r^*_t and will purchase through direct channel j in the end.

With a slight abuse of the notation, denote $\widetilde{\Pi}_{it} = \widetilde{\Pi}_i(\mathbf{x}, t)$ and $\widetilde{\Pi}_{jt} = \widetilde{\Pi}_j(\mathbf{x}, t)$. We next characterize the equilibrium pricing strategy when seller i is the opaque seller (i.e., $\widetilde{\Pi}_{it} \leq \widetilde{\Pi}_{jt}$). For seller i's optimal pricing response given p_{jt}:

- First consider the case when $p_{jt} \geq v^r_{jt}$. If $p_{it} \geq v^r_{it}$, customers with $v_i \geq v^r_{it}$ will purchase through the NYOP channel, and those with $v_i < v^r_{it}$ are not accepted by the NYOP channel and cannot afford either direct channel, hence will left empty-handed. Seller i's expected profit is $\Pi_1(\mathbf{x}, t) = \Pi_1(x_1 - 1, x_2, t-1)\dfrac{1 - v^r_{it}}{1 - \bar{v}_t} + \Pi_1(x_1, x_2, t-1)\dfrac{v^r_{it} - \bar{v}_t}{1 - \bar{v}_t}$. If $p_{it} < v^r_{it}$, bids from all customers will be rejected. The only realized sales are through direct channel i. The expected profit for seller i is $\Pi_1(\mathbf{x}, t) = [\, p_{it} + \Pi_1(x_1 - 1, x_2, t-1)\,] \dfrac{1 - p_{it}}{1 - \bar{v}_t} + \Pi_1(x_1, x_2, t-1)\dfrac{p_{it} - \bar{v}_t}{1 - \bar{v}_t}$. Apparently, it is optimal for seller i to set $p_{it} = v^r_{it} - \epsilon$ for some small $\epsilon > 0$ in this scenario.
- For the case where $p_{jt} < v^r_{jt}$, if $p_{it} > p_{jt} + \bar{v}_t$, then all bids will be rejected and customers with $v_{jt} \geq p_{jt}$ will buy through direct channel j in the end. Seller i's expected profit is $\Pi_1(\mathbf{x}, t) = \Pi_1(x_1, x_2 - 1, t-1)\dfrac{1 - p_{jt} - \bar{v}_t}{1 - \bar{v}_t} + \Pi_1(x_1, x_2, t-1)\dfrac{p_{jt}}{1 - \bar{v}_t}$. If $p_{it} < p_{jt} + \bar{v}_t$, all bids will be rejected and customers with $v_{it} \geq p_{it}$ will buy through direct channel i in the end. Seller i's expected profit is $\Pi_1(\mathbf{x}, t) = [\, p_{it} + \Pi_1(x_1 - 1, x_2, t-1)\,] \dfrac{1 - p_{it}}{1 - \bar{v}_t} + \Pi_1(x_1, x_2, t-1)\dfrac{p_{it} - \bar{v}_t}{1 - \bar{v}_t}$. Comparing the two options, it is optimal for seller i to set $p_{it} = p_{jt} + \bar{v}_t - \epsilon$ for some small $\epsilon > 0$ if $p_{jt} + \bar{v}_t > \widetilde{\Pi}_{it}$.

Overall, $p^*_{it}(p_{jt}) = \min\{\max\{\widetilde{\Pi}_{it}, p_{jt} + \bar{v}_t\}, v^r_{it}\}$. Similarly for seller j, it can be verified that $p^*_{jt}(p_{it}) = \min\{\max\{\widetilde{\Pi}_{jt}, p_{it} - \bar{v}_t\}, v^r_{jt}\}$. The same pair of response functions hold when seller j is the opaque product provider (i.e., $\widetilde{\Pi}_{it} \geq \widetilde{\Pi}_{jt}$). In either case, sales are realized through one direct channel only, and no sales will be through the NYOP channel.

(ii) For the horizontal differentiation case, consider that $\{\alpha_i v_i + \alpha_j v_j = \bar{v}_t\}$ for some $\bar{v}_t > 0$. $\mathbf{H}_{DC}(\mathbf{v})$ suggests that there are *multiple* types of channel a customer can possibly end up with, depending on her valuation realization (v_{it}, v_{jt}).

Denote $v^*_t = r^*_t + \dfrac{G(r^*_t)}{g(r^*_t)}$, where $r^*_t = \min\{\widetilde{\Pi}_{it}, \widetilde{\Pi}_{jt}\}$ is the reservation price in period t. Then, if $\bar{v}_t \geq v^*_t$, all customers will bid above r^*_t when $v_{it} - p_{it} < 0$ and $v_{jt} - p_{jt} < 0$. We next show that if $\bar{v}_t \geq v^*_t$, then there exists some conditions under which $\Omega_O > 0$ with equilibrium pricing (p^*_{it}, p^*_{jt}).

We start with analyzing seller i's optimal pricing response given p_{jt}:

- If $p_{it} \leq v^r_t - p_{jt}$, then $\Omega_i = \alpha_j \dfrac{v^r_t - \alpha_i p_{it} + \alpha_i p_{jt}}{v^r_t}$ and $\Omega_0 = \Omega_O = 0$. As analyzed in the proof of Theorem 3,

$$\frac{\partial \Pi_i}{\partial p_{it}} = \Omega_i - \left(\widetilde{\Pi_i} - p_{it}\right)\frac{\partial \Omega_i}{\partial p_{it}} - \Delta\Pi_i \frac{\partial \Omega_0}{\partial p_{it}} \tag{A11}$$

where $\Delta\Pi_i = \Pi_i(x_i, x_j - 1, t-1) - \Pi_i(x_i, x_j, t-1)$. Then,

$$\begin{aligned}\frac{\partial \Pi_i}{\partial p_{it}} &= \alpha_j \frac{v_t^r - \alpha_i p_{it} + \alpha_i p_{jt}}{v_t^r} - \left(p_{it} - \widetilde{\Pi_i}\right)\frac{\alpha_i \alpha_j}{v_t^r} = \frac{\alpha_i \alpha_j}{v_t^r}\left(\frac{v_t^r}{\alpha_i} - 2p_{it} + p_{jt} + \widetilde{\Pi}_{it}\right) \\ &\geq \frac{\alpha_i \alpha_j}{v_t^r}\left(\frac{v_t^r}{\alpha_i} - 2v_t^r + 3p_{jt} + \widetilde{\Pi}_{it}\right)\end{aligned}$$

- If $p_{it} > v_t^r - p_{jt}$, then $\Omega_i = \frac{v_t^r - \alpha_i p_{it}}{v_t^r}$, $\Omega_j = \frac{v_t^r - \alpha_j p_{jt}}{v_t^r}$, $\Omega_O = \frac{\alpha_i p_{it} + \alpha_j p_{jt} - v_t^r}{v_t^r}$ and $\Omega_0 = 0$.

Thus,

$$\frac{\partial \Pi_i}{\partial p_{it}} = \frac{v_t^r - \alpha_i p_{it}}{v_t^r} - \left(p_{it} - \widetilde{\Pi_i}\right)\frac{\alpha_i}{v_t^r} = \frac{\alpha_i}{v_t^r}\left(\frac{v_t^r}{\alpha_i} - 2p_{it} + \widetilde{\Pi}_{it}\right)$$

The first order derivative is greater than zero if $p_{it} < \frac{\frac{v_t^r}{\alpha_i} + \widetilde{\Pi}_{it}}{2}$.

The same analysis can be done for seller j. It can be verified that the equilibrium is

$$(p_{it}^*, p_{jt}^*) = \left(\frac{\frac{v_t^r}{\alpha_i} + \widetilde{\Pi}_{it}}{2}, \frac{\frac{v_t^r}{\alpha_j} + \widetilde{\Pi}_{jt}}{2}\right),$$

at which $\Omega_O = \frac{\alpha_i \widetilde{\Pi}_{it} + \alpha_j \widetilde{\Pi}_{jt}}{2v_t^r} > 0$ if one of $\widetilde{\Pi}_{it}$ and $\widetilde{\Pi}_{jt}$ is nonzero. Thus the NYOP channel earns non-zero profit when $v_t^r \leq \bar{v}_t$ and $\max\{\widetilde{\Pi}_{it}, \widetilde{\Pi}_{jt}\} > 0$.

□